Application of Nonlinear Moving Horizon Estimation

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TWMCC Meeting
University of Wisconsin–Madison

September 28, 2000

Outline

- Introduction
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    Track Radioactive Sources within a Facility
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  - Extended Kalman Filter
  - Iterative Extended Kalman Filter
  - Second Order Extended Kalman Filter
  - Moving Horizon Algebraic Estimator
- Estimator Comparison
- Conclusions

Facility Monitoring

- Monitor Radioactive Material within a Facility
  - ⋆ track location using detector measurements
- Facility Monitoring Experiment
  - ⋆ four radiation detectors located within the room
  - ⋆ one radioactive source moved to 17 different locations
  - ⋆ source stationary at each location for $\approx$ 3 min

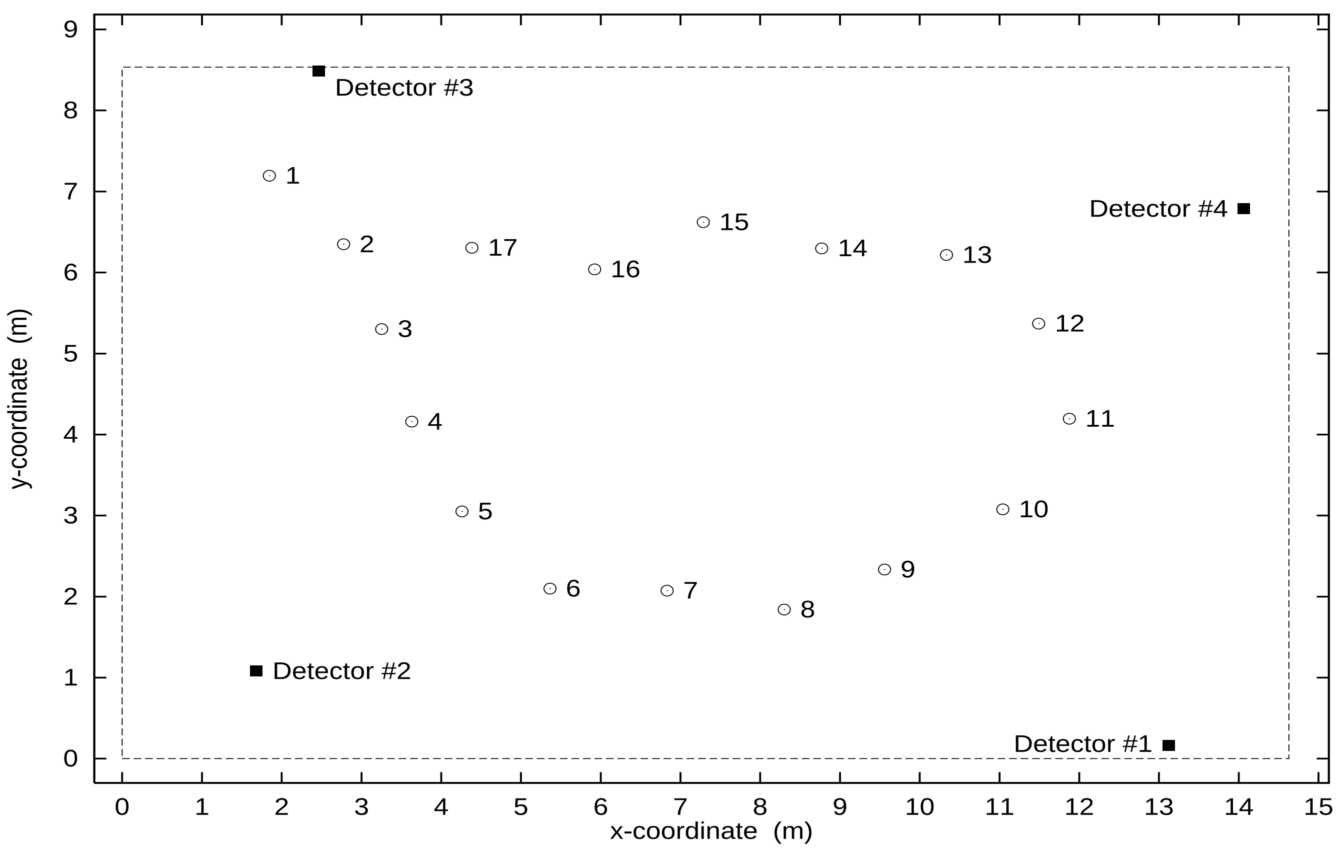


Figure 1: Detector and source locations.

Detector Measurement Data

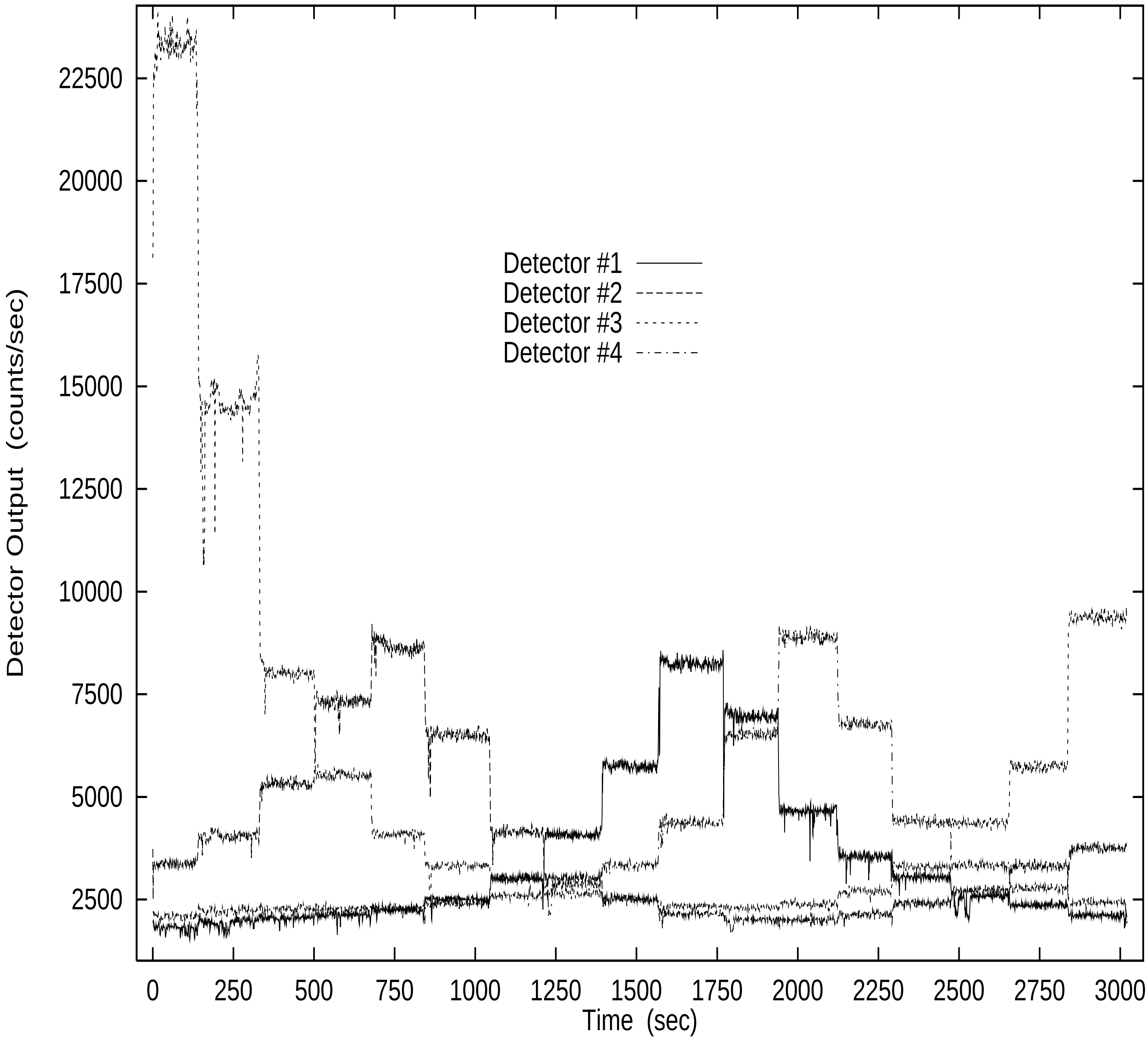


Figure 2: Detector measurements.

Tracking System Model

- Dynamic Model (random walk process)

$$\dot{\boldsymbol{s}} = 0 + \boldsymbol{\omega}, \qquad \boldsymbol{s} = [x\ y\ z\ \mathcal{B}]^T$$

  - ⋆ x–y–z: coordinates of the source (m)
  - ⋆ $\mathcal{B}$: background radiation (counts/sec)
  - ⋆ $\boldsymbol{\omega}$: independent Gaussian disturbance vector

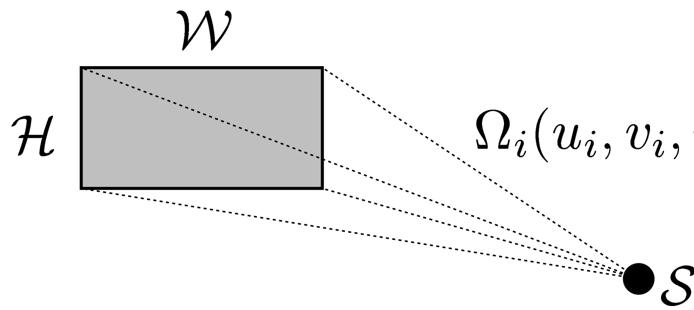
- Detector Model

$$\mathcal{M}_i = \frac{\Omega_i\,\mathcal{S}\,\epsilon\,\mathcal{F} + \mathcal{B}}{1 + \tau\,\Omega_i\,\mathcal{S}\,\epsilon\,\mathcal{F}} + \nu_i, \quad i = 1, \ldots, 4$$

  - ⋆ $\mathcal{M}_i$: output of detector i (counts/sec)
  - ⋆ Ω_i: view factor of detector i
  - ⋆ $\mathcal{S}$: source strength (counts/sec)
  - ⋆ $\mathcal{B}$: background radiation (counts/sec)
  - ⋆ ϵ: detector efficiency, τ: detector dead time
  - ⋆ $\mathcal{F}$: correction factor
  - ⋆ $\boldsymbol{\nu}$: independent Poisson measurement noise vector

View Factor Determination

- Solid Angle Subtended by Detector Surface



⋆ u_i–v_i–w_i: detector i centered source coordinates
⋆ $\mathcal{W}$: detector width $\mathcal{H}$: detector height

$$\Omega_i(u_i, v_i, w_i) = \tan^{-1}\left(\frac{u_i\,v_i}{|w_i|\sqrt{u_i^2+v_i^2+w_i^2}}\right)$$
$$-\tan^{-1}\left(\frac{(u_i-\mathcal{W})\,v_i}{|w_i|\sqrt{(u_i-\mathcal{W})^2+v_i^2+w_i^2}}\right)$$
$$-\tan^{-1}\left(\frac{u_i\,(v_i-\mathcal{H})}{|w_i|\sqrt{u_i^2+(v_i-\mathcal{H})^2+w_i^2}}\right)$$
$$+\tan^{-1}\left(\frac{(u_i-\mathcal{W})\,(v_i-\mathcal{H})}{|w_i|\sqrt{(u_i-\mathcal{W})^2+(v_i-\mathcal{H})^2+w_i^2}}\right)$$

Coordinate Transformation

- Transform Room-centered Coordinates into Detector-centered Coordinates for Ω_i Determination

$$\begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} = \begin{pmatrix} \cos \mathcal{A}_i & 0 & \sin \mathcal{A}_i & \begin{matrix} \mathcal{T}_{x,i} \cos \mathcal{A}_i + \\ \mathcal{T}_{z,i} \sin \mathcal{A}_i \end{matrix} \\ 0 & 1 & 0 & \mathcal{T}_{y,i} \\ -\sin \mathcal{A}_i & 0 & \cos \mathcal{A}_i & \begin{matrix} \mathcal{T}_{z,i} \cos \mathcal{A}_i - \\ \mathcal{T}_{x,i} \sin \mathcal{A}_i \end{matrix} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

  - ⋆ $\mathcal{T}_{x,i}$: x-coordinate translation for detector i
  - ⋆ $\mathcal{T}_{y,i}$: y-coordinate translation for detector i
  - ⋆ $\mathcal{T}_{z,i}$: z-coordinate translation for detector i
  - ⋆ $\mathcal{A}_i$: angle of rotation for detector i

- Source Strength and Background are Invariant under this Transformation

Least Squares Estimator

- Minimize Nonlinear Least Squares Objective

$$\boldsymbol{d}_k^* = \arg\min_{\boldsymbol{d}_k} \left(\sum_{i=1}^{4} w_i^m \left(\mathcal{D}_i(k) - \mathcal{M}_i\big(\hat{\boldsymbol{s}}(k-1) + \boldsymbol{d}(k)\big) \right)^2 + \sum_{j=1}^{4} w_j^d \, d_j^2(k) \right)$$

subject to:

$$\begin{array}{c} -\mathcal{R}_j \leq d_j(k) \leq \mathcal{R}_j, \\ \mathcal{L}_j \leq \hat{s}_j(k-1) + d_j(k) \leq \mathcal{U}_j, \end{array} \quad j = 1, \ldots, 4$$

  - ⋆ $\mathcal{D}_i(k)$: measured detector i count rate at sample k
  - ⋆ $\mathcal{M}_i\big(\hat{\boldsymbol{s}}(k-1) + \boldsymbol{d}(k)\big)$: predicted detector i count rate
  - ⋆ $\boldsymbol{d}(k)$: $[\delta x(k)\ \delta y(k)\ \delta z(k)\ \delta \mathcal{B}(k)]^T$
  - ⋆ prediction horizon of 1 (no state dynamics)

- Least Squares Weights
  - ⋆ $w_j^d = 1$: state disturbance variance
  - ⋆ $w_j^m = (\gamma_j^2 \mathcal{M}_j(\boldsymbol{s}(k)))^{-1}$: sensor noise variance
  - ⋆ $\gamma_j = 10$: dispersion (ratio of variance to mean)

Tracking Error Results

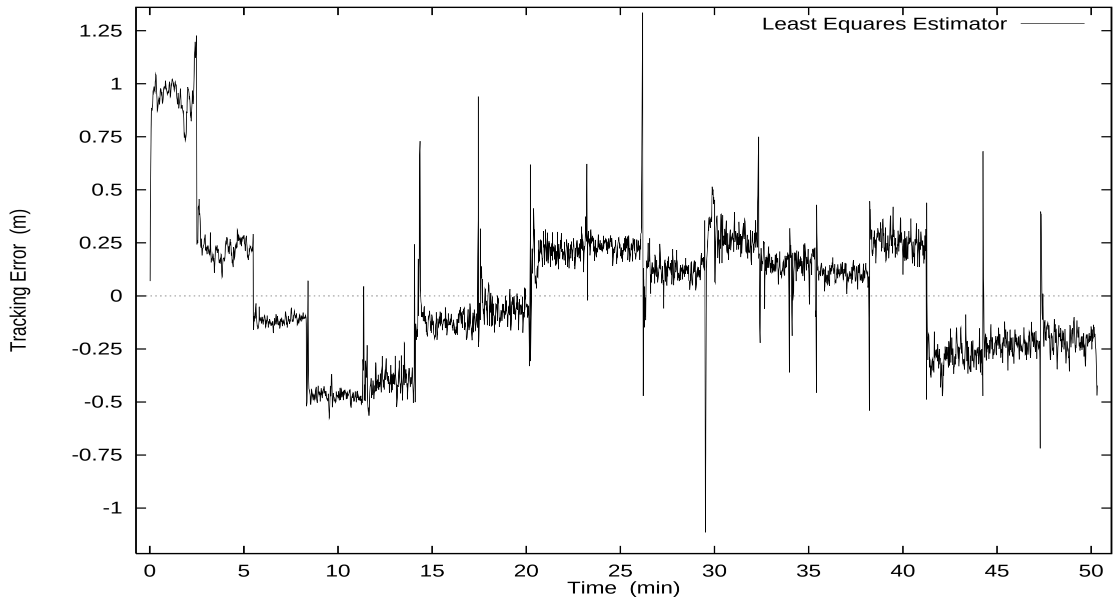


Figure 3: x–direction tracking error.

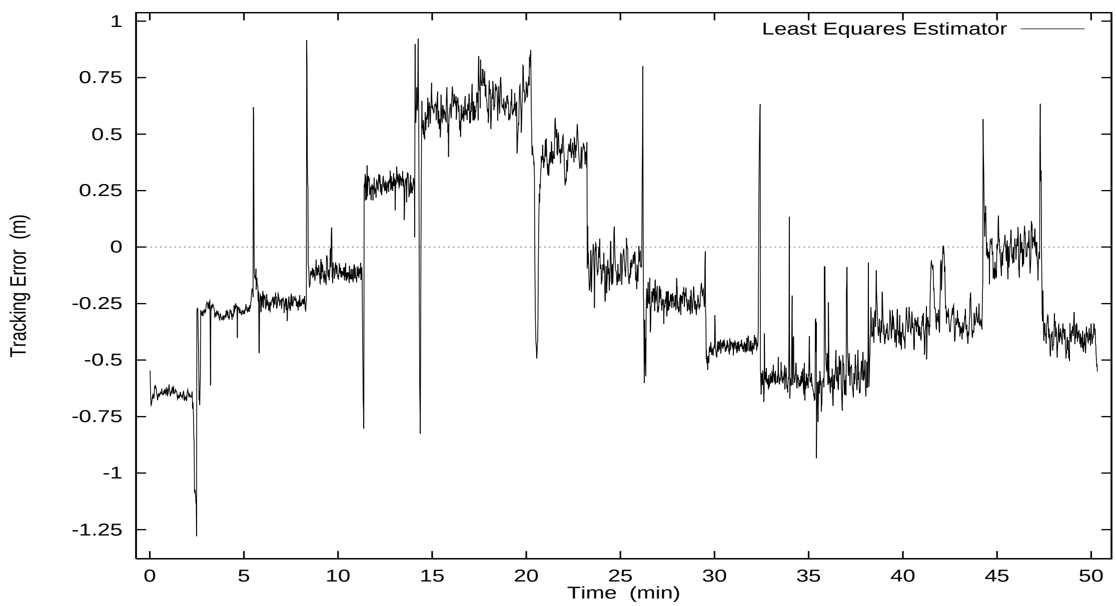


Figure 4: y–direction tracking error.

Least Squares Estimation

- Tracking Results
  - ⋆ mean tracking error: x–0.25 m, y–0.36 m
  - ⋆ norm tracking error: x–17.8 m, y–23.0 m
  - ⋆ solution in real time (FSQP algorithm)

- Theoretical Criticism
  - ⋆ sub-optimal estimator
    - ∗ first order accurate approximation
    - ∗ approaches optimal tracking error (Monte Carlo Simulation)
    - ∗ second order accurate estimator in progress

- Practical Criticism
  - ⋆ requires implementation of on-line optimization
  - ⋆ nonlinear, constrained optimization software
  - ⋆ relatively fast computer (UltraSparc 20)
  - ⋆ why not implement a recursive filter (*e.g.* EKF)?

Extended Kalman Filter

- Filter Equations

$$
\begin{aligned}
\hat{\boldsymbol{s}}(k) &= \hat{\boldsymbol{s}}(k-1) + \boldsymbol{L}(k)\left(\boldsymbol{\mathcal{D}}(k) - \boldsymbol{\mathcal{M}}(k)\right) \\
\boldsymbol{L}(k) &= \boldsymbol{P}(k)\boldsymbol{\mathcal{G}}^T(k)\left(\boldsymbol{\mathcal{G}}(k)\boldsymbol{P}(k)\boldsymbol{\mathcal{G}}^T(k) + \boldsymbol{R}\right)^{-1} \\
\boldsymbol{R} &= \text{diag}[\gamma^2 \boldsymbol{\mathcal{M}}(k)]
\end{aligned}
$$

- Linearized Measurement Function

$$
\boldsymbol{\mathcal{G}} = \begin{pmatrix}
\frac{\partial \mathcal{M}_1}{\partial x} & \frac{\partial \mathcal{M}_1}{\partial y} & \frac{\partial \mathcal{M}_1}{\partial z} & \frac{\partial \mathcal{M}_1}{\partial \mathcal{B}} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{\partial \mathcal{M}_4}{\partial x} & \frac{\partial \mathcal{M}_4}{\partial y} & \frac{\partial \mathcal{M}_4}{\partial z} & \frac{\partial \mathcal{M}_4}{\partial \mathcal{B}}
\end{pmatrix}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{M}_i}{\partial x} &= \frac{\partial \mathcal{M}_i}{\partial \Omega_i}\left(\frac{\partial \Omega_i}{\partial u_i}\frac{\partial u_i}{\partial x} + \frac{\partial \Omega_i}{\partial v_i}\frac{\partial v_i}{\partial x} + \frac{\partial \Omega_i}{\partial w_i}\frac{\partial w_i}{\partial x}\right) \\
\frac{\partial \mathcal{M}_i}{\partial y} &= \frac{\partial \mathcal{M}_i}{\partial \Omega_i}\left(\frac{\partial \Omega_i}{\partial u_i}\frac{\partial u_i}{\partial y} + \frac{\partial \Omega_i}{\partial v_i}\frac{\partial v_i}{\partial y} + \frac{\partial \Omega_i}{\partial w_i}\frac{\partial w_i}{\partial y}\right) \\
\frac{\partial \mathcal{M}_i}{\partial z} &= \frac{\partial \mathcal{M}_i}{\partial \Omega_i}\left(\frac{\partial \Omega_i}{\partial u_i}\frac{\partial u_i}{\partial z} + \frac{\partial \Omega_i}{\partial v_i}\frac{\partial v_i}{\partial z} + \frac{\partial \Omega_i}{\partial w_i}\frac{\partial w_i}{\partial z}\right)
\end{aligned}
$$

Tracking Error Results

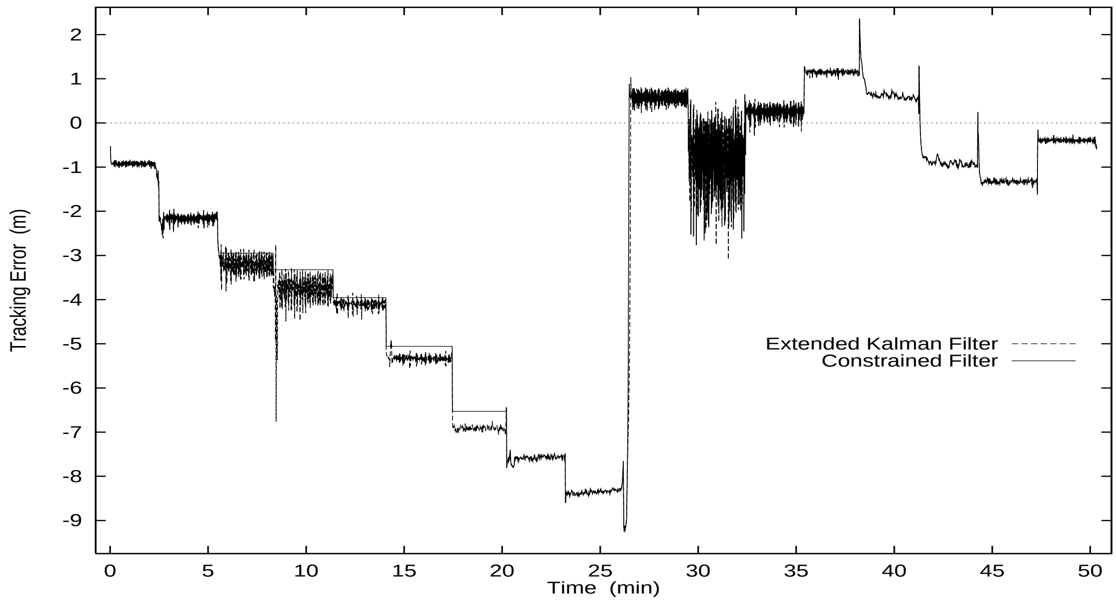


Figure 5: x–direction tracking error.

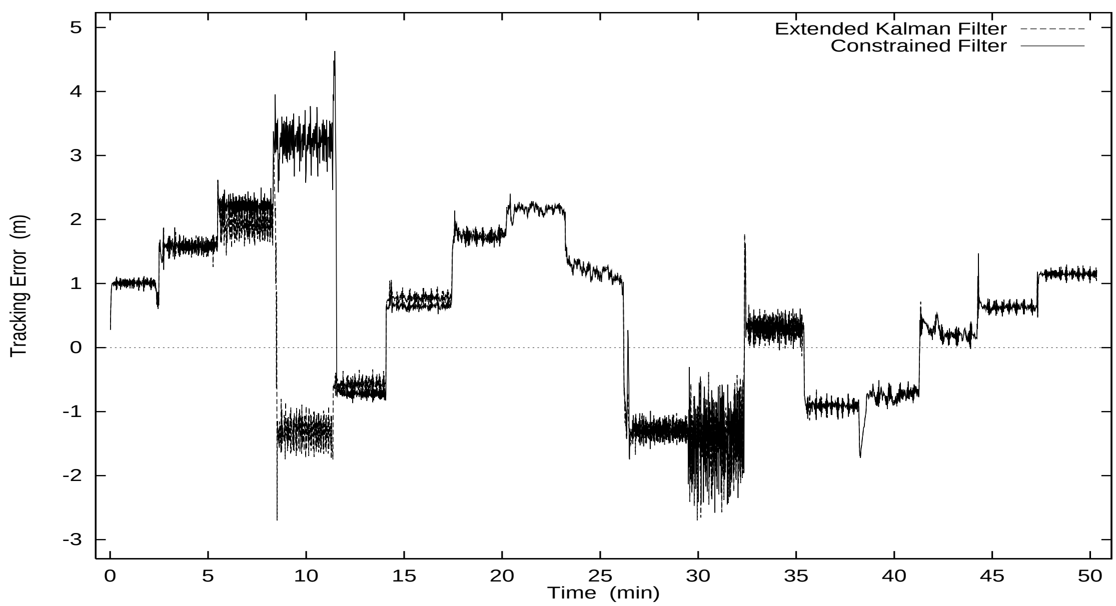


Figure 6: y–direction tracking error.

Iterative Extended Kalman Filter

- Iterate Extended Kalman Filter
  - ⋆ repeat linearization of measurement function
  - ⋆ improved function approximation at each iterate
  - ⋆ converge to a better state estimate
  - ⋆ published examples – significant improvement

- Filter Equations

$$
\begin{aligned}
\hat{\boldsymbol{s}}(k,i+1) &= \hat{\boldsymbol{s}}(k-1) + \boldsymbol{L}(k,i)\Big(\boldsymbol{\mathcal{D}}(k) - \boldsymbol{\mathcal{M}}(k,i) \\
&\quad -\boldsymbol{\mathcal{G}}(k,i)\big(\hat{\boldsymbol{s}}(k-1) - \hat{\boldsymbol{s}}(k,i)\big)\Big) \\
\boldsymbol{R} &= \mathrm{diag}[\gamma^2 \boldsymbol{\mathcal{M}}(k,i)] \\
\boldsymbol{\mathcal{M}}(k,i) &: \text{Computed using } \hat{\boldsymbol{s}}(k,i) \\
\boldsymbol{\mathcal{G}}(k,i) &: \text{Computed using } \hat{\boldsymbol{s}}(k,i) \\
\boldsymbol{L}(k,i) &: \text{Computed using } \boldsymbol{\mathcal{G}}(k,i)
\end{aligned}
$$

  - ⋆ compensate for measurement function nonlinearity
  - ⋆ 10 iteration limit

Tracking Error Results

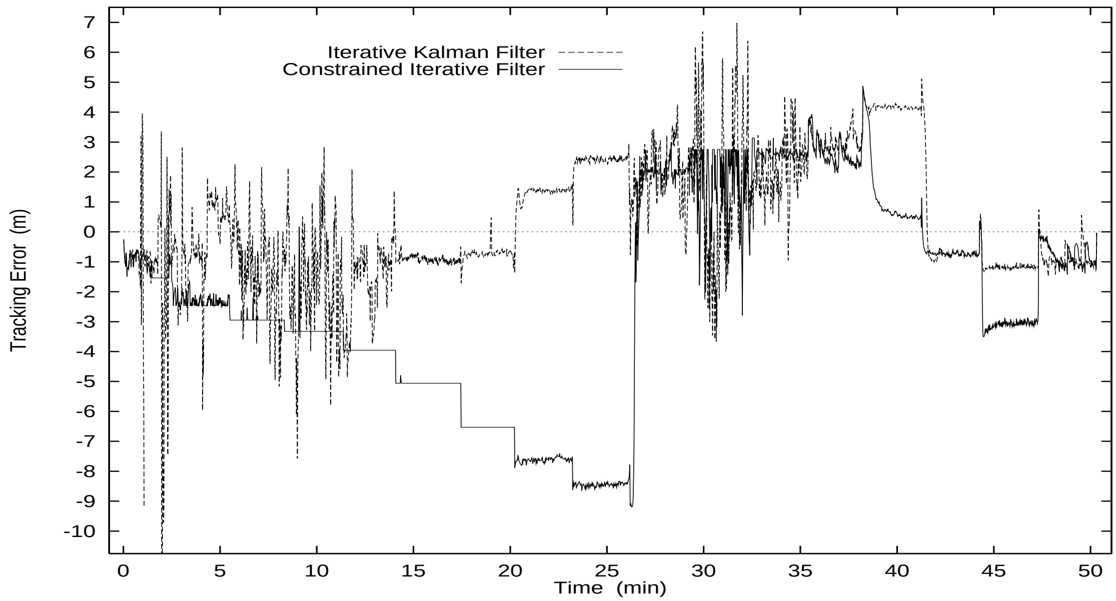


Figure 7: x–direction tracking error.

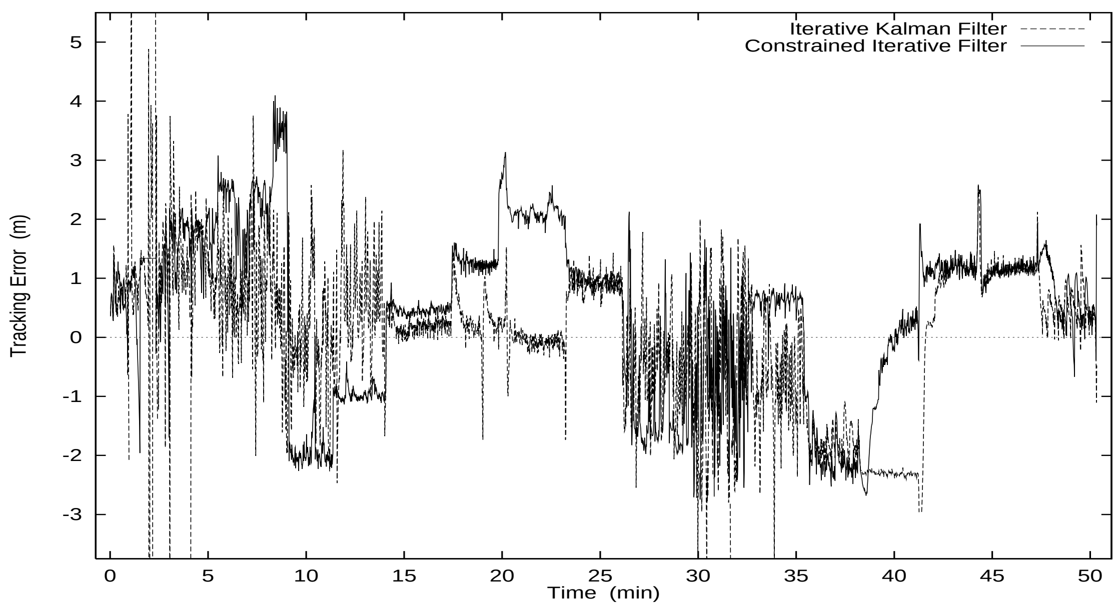


Figure 8: y–direction tracking error.

Second Order Extended Kalman Filter

- Second Order Filter
  - ⋆ first order approximation is not sufficient
  - ⋆ take second order terms into account
  - ⋆ published examples – inferior to iterative filter

- Filter Equations

$$\hat{\boldsymbol{s}}(k) = \hat{\boldsymbol{s}}(k-1) + \boldsymbol{L}(k)\Big(\boldsymbol{\mathcal{D}}(k) - \boldsymbol{\mathcal{M}}(k) - \boldsymbol{\Pi}(k)\Big)$$

$$\boldsymbol{L}(k) = \boldsymbol{P}(k)\boldsymbol{\mathcal{G}}^T(k)\left(\boldsymbol{\mathcal{G}}(k)\boldsymbol{P}(k)\boldsymbol{\mathcal{G}}^T(k) + \boldsymbol{R} + \boldsymbol{\Gamma}(k)\right)^{-1}$$

$$\boldsymbol{R} = \text{diag}[\gamma^2 \boldsymbol{\mathcal{M}}(k)]$$

$$\boldsymbol{\Pi}(k) = \frac{1}{2}\sum_{j=1}^{4} \boldsymbol{e}_j \text{trace}\left[\boldsymbol{G}_j(k)\boldsymbol{P}(k)\right]$$

$$\boldsymbol{\Gamma}(k) = \frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4} \boldsymbol{e}_j \boldsymbol{e}_i^T \text{trace}\left[\boldsymbol{G}_i(k)\boldsymbol{P}(k)\boldsymbol{G}_j(k)\boldsymbol{P}(k)\right]$$

$$\boldsymbol{G}_i(k) = \left.\frac{\partial^2 \mathcal{M}_i}{\partial \boldsymbol{s}^2}\right|_{\boldsymbol{s}=\boldsymbol{s}(k)}$$

Tracking Error Results

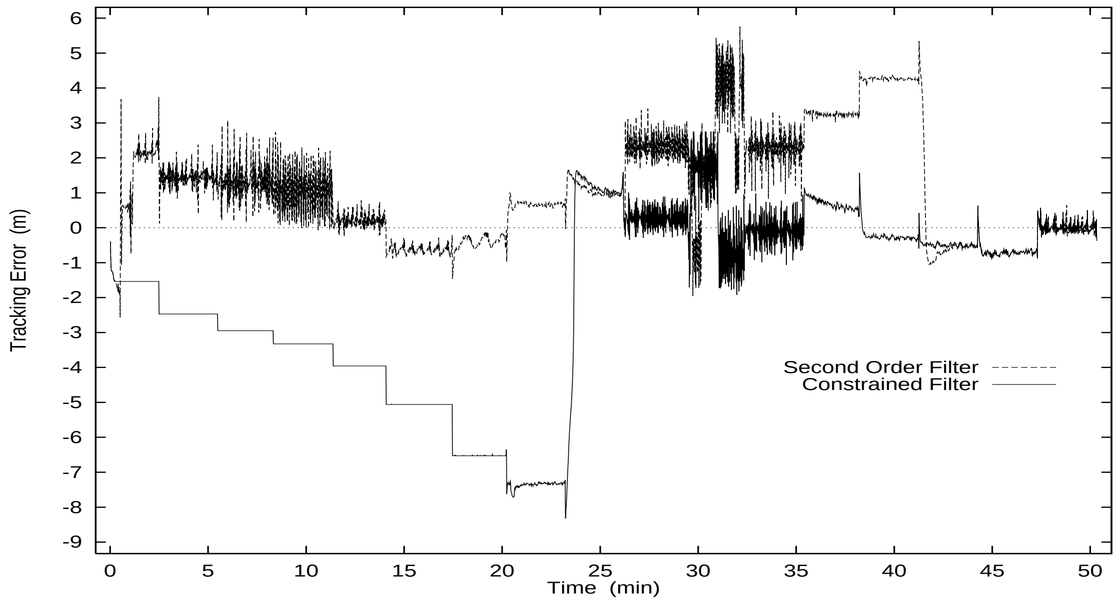


Figure 9: x–direction tracking error.

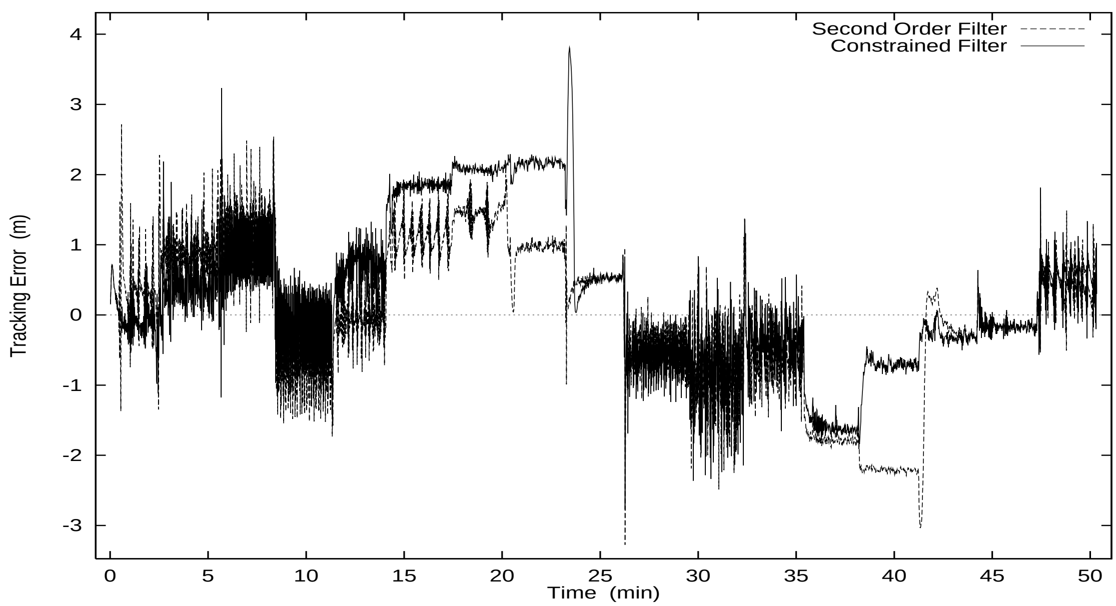


Figure 10: y–direction tracking error.

Estimator Comparison

Filter	*2-norm error*		*mean error*	
	x (m)	y (m)	x (m)	y (m)
1st order	216.9	69.1	4.63	1.28
1st order(c)	210.4	81.1	3.26	1.35
Iterative	120.7	74.3	2.56	1.31
Iterative(c)	223.3	80.9	4.71	1.34
2nd order	105.9	59.1	2.13	1.01
2nd order(c)	177.2	62.4	2.90	1.12
least squares(c)	17.8	23.8	0.25	0.36

Table 1: Tracking errors. (c) = constrained filter.

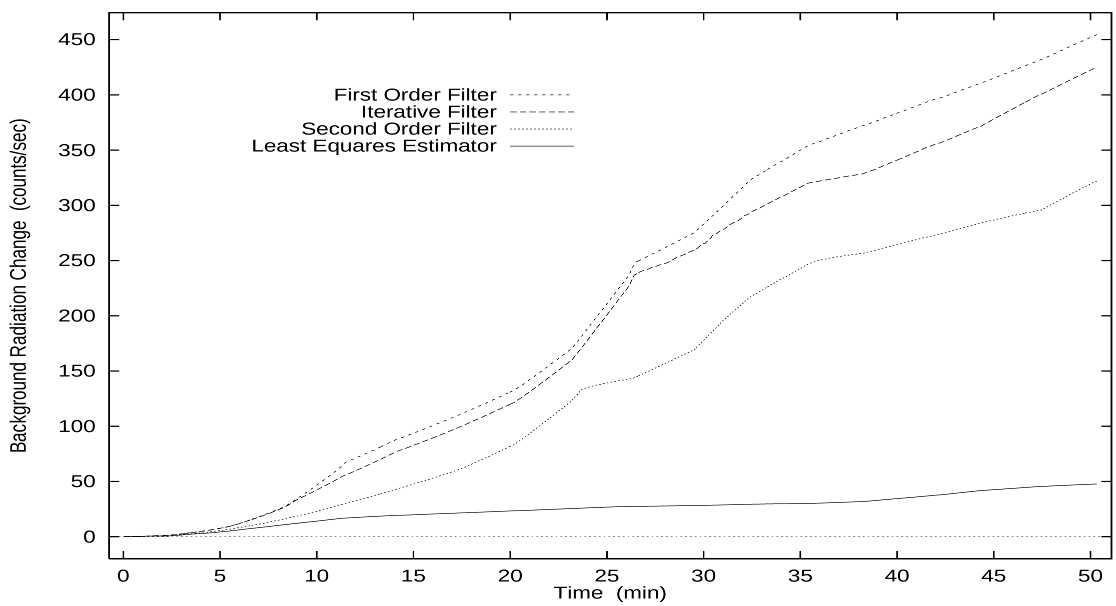


Figure 11: Estimated background radiation change.

Moving Horizon Algebraic Estimator

- Solve System of Equations for Estimation Problem

$$\boldsymbol{\mathcal{D}}(k) - \boldsymbol{\mathcal{M}}\Big(\hat{\boldsymbol{s}}(k-1) + \boldsymbol{d}(k)\Big) = \boldsymbol{0}$$

  - ⋆ prediction horizon of 1 (no state dynamics)
  - ⋆ no noise model present
  - ⋆ Newton's method to solve nonlinear system

$$\begin{aligned}\boldsymbol{d}(k, i+1) \quad &= \quad \boldsymbol{d}(k, i) + \boldsymbol{\mathcal{G}}(k, i)^{-1} \\ &\quad \Big(\boldsymbol{\mathcal{D}}(k) - \boldsymbol{\mathcal{M}}\big(\hat{\boldsymbol{s}}(k-1) + \boldsymbol{d}(k, i)\big)\Big)\end{aligned}$$

- Tracking Error Results

  - ⋆ will not converge without a line search
  - ⋆ Newton direction is poor search direction
  - ⋆ computational time exceeds least squares estimator
  - ⋆ performance comparable to 1st order EKF

- Poor Estimation Algorithm for this Application

Conclusions

- Least Squares Estimation
  - Excellent Tracking Results
  - Solved in Real Time
- Recursive Filters
  - Very Poor Tracking Results
  - Nonlinearity in View Factor Relationship
  - Computational Time Approaches Optimization Time
- Future Work
  - Multiple Source Tracking
  - Statistical Detection of Source Removal
  - Optimal Estimator Formulation
- Acknowledgments
  - James Howse & Larry Ticknor (collaborators)
  - US Department of Energy (funding)