Performance Monitoring of the LQG System

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1 Introduction

There are many reasons that a loop may not live up to its expectations including : poor tuning, model mismatch, or unmeasured disturbances. When compared to using plant tests to diagnose problems, performing these analyses using only normal operating data is very attractive economically. So far this project has focused primarily on poor tuning, but will be extended to include some of the aforementioned components.

2 Objectives

Detection of the following problems using only ordinary process data

- Poor Tuning
- Model Mismatch
- Deterministic disturbances

Eventual extensions to :

- Model Predictive Control (MPC)
- Non-Gaussian Disturbances

3 Motivating Example

As a starting point, we assume a process operating below as shown in Figure 1 with good tracking, but agressive control action. This simulation assumes imperfect knowledge of the noise variances. As a fix to slow the control action, the constraints on control moves could be tightened at the expense of tracking as shown in Figure 2.

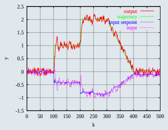

Figure 1:Poor variance estimate

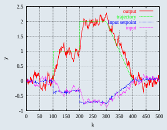

Figure 2:Incorrect control knob

The better control strategy would be to detect poor variance estimates and correct as shown in Figure 3. As shown, the control action is more conservative while maintaining good setpoint tracking.

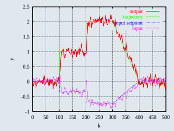

Figure 3:Better tuning performance

Initial research focuses on the unforced system

$$
\begin{aligned}
x_{k+1} &= Ax_k + w_k \\
y_k &= Cx_k + d_k + v_k
\end{aligned}
$$

where w, and v are white noise terms defined by :

$$w \sim N(0,Q) \qquad v \sim N(0,R)$$

$$A = \begin{bmatrix} 0.73262 & -0.086107 \\ 0.16221 & 0.99094 \end{bmatrix} \quad Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} \quad R = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$$

Can Q and R be recovered based strictly on the normal input/output information provided from the process?

4 Filtering/Smoothing

The optimal estimate for x given from the Kalman filter and measurement correction are given by

$$
\begin{aligned}
\hat{x}_{\overline{k+1}} &= A\hat{x}_k \\
\hat{x}_k &= \hat{x}_{\overline{k}} + L_x[y_k - (C\hat{x}_k)]
\end{aligned}
$$

Estimates of the noise terms from simple rearrangement

$$\hat{v}_k = y_k - C\hat{x}_k \hat{w}_k = \hat{x}_{k+1} - A\hat{x}_k$$

$$E\begin{bmatrix} \hat{w}_k\hat{w}_k' & \hat{w}_k\hat{v}_k' \\ \hat{v}_k\hat{w}_k' & \hat{v}_k\hat{v}_k' \end{bmatrix} = \begin{bmatrix} L(CPC'+R)L' & (CPC'+R)L' \\ L(CPC'+R) & CPC'+R \end{bmatrix}$$

Here P represented the solution to the steady state Riccati equation. This matrix gives the estimates of the variances generated by the model. This information will be compared to the actual data generated by the system. As shown in Figure 4, when the covariance estimates are correct, the probability distributions for the model and data conincide nicely.

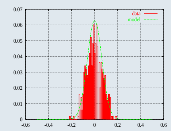

Figure 4:Good estimate of sensor noise

However, if the estimates are too low or too high (shown respectively below) it is easy to detect the mismatch.

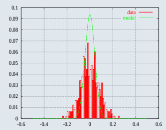

Figure 5:Variance estimate too low

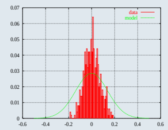

Figure 6:Variance estimate too high

Due to the formulation of the problem, however, an incorrect estimate in the state noise or measurement noise will show up in both probability distributions. Similarly, the smoothing problem can be defined as followed in an attempt to recover the desired variances.

$$\Phi = \begin{bmatrix} x_0 & v_0 & w_0 & x_1 & \cdots \end{bmatrix} \begin{bmatrix} & & \\ & H & \\ & & \end{bmatrix} \begin{bmatrix} x_0 \\ v_0 \\ w_0 \\ x_1 \\ \vdots \end{bmatrix} + \begin{bmatrix} x_0 & v_0 & w_0 & x_1 & \cdots \end{bmatrix} \begin{bmatrix} 0 \\ C^T R^{-1} y_0 \\ 0 \\ C^T R^{-1} y_1 \\ \vdots \end{bmatrix}$$

$$
\begin{gathered}
\min \Phi(x) = x^T H x \\
\text{st } Dx = d
\end{gathered}
$$

$$E\begin{bmatrix} xx' & x\lambda' \\ \lambda x' & \lambda\lambda' \end{bmatrix} = \begin{bmatrix} 2H & -D^T \\ D & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 \\ 0 & E(dd') \end{bmatrix} \left(\begin{bmatrix} 2H & -D^T \\ D & 0 \end{bmatrix}^{-1} \right)^T$$

The necessary variances can be extracted from the solution. However, performing the same analysis that was used on the filtering technique reveals that the smoothing problem exhibits the same coupling of effects that makes reconstruction of the noise statistics impossible.

5 Probabilistic Least-Squares

The output data is treated as a random variable itself (i.e. a transformation of the forcing random variables) as shown:

$$E\begin{bmatrix} y_0y0' & * & * \\ y_1y0' & y_1y_1' & * \\ y_2y0' & y_2y_1' & y_2y_2' \end{bmatrix} =$$

$$\begin{bmatrix} CQ_0C' + R & * & * \\ CAQ_0C' & CAQ_0A'C' + CQC' + R & * \\ CAAQ_0C' & CAAQ_0A'C' + CAQC' & CAAQ_0A'A'C' + CAQA'C' + CQC' + R \end{bmatrix}$$

The upper diagonal terms of the matrix are excluded due to the symmetry of the matrix. Generalization of the covariance terms follows and can be used to generalize the method to a variable window size N.

$$E\left[y_j y_k'\right] = CA^j Q_0 (A')^k C^k + \sum_{h=1}^{k} CA^{j-k+h-1} Q (A')^{h-1} C' + \begin{cases} 0 & j \neq k \\ R & j = k \end{cases}$$

To generate the actual covariances from the data, a number of replications are performed. Instead of using multiple experiments, a window (fixed size N) of data can be used and slid through a long string of data to generate the proper outputs.

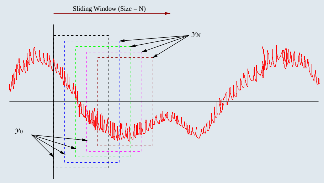

Figure 7:Sliding Data Window

In this formulation of the problem, the Q_0 term represents the solution to the Riccati equation as opposed to the initial variance of the states.

$$\begin{bmatrix} & \vdots & \\ \cdots & A & \cdots \\ & \vdots & \end{bmatrix} \begin{bmatrix} Q_0 & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} & \vdots & \\ \cdots & C & \cdots \\ & \vdots & \end{bmatrix} = \begin{bmatrix} y_0y_0' & * & * \\ \vdots & \ddots & * \\ y_jy_0' & \cdots & y_jy_k' \end{bmatrix}$$

Each of the matrices can be stacked to reformulate the problem as the standard least-squares program $Ax = b$. Once computed, the solution vector can be unstacked into the matrix solution form.

$$\begin{bmatrix} & \vdots & \\ \cdots & A & \cdots \\ & \vdots & \end{bmatrix}_{stacked} \begin{bmatrix} Q_0 \\ Q \\ R \end{bmatrix}_{stacked} = \begin{bmatrix} y_0y_0' \\ \vdots \\ y_jy_k' \end{bmatrix}_{stacked}$$

Eventually, these equations will be solved using semi-definite programming (SDP) which should greatly improve calculation efficiency.

6 Results

As stated, the objective of this project is to compute the noise statistics for use in the LQ filter. Figure 8 below demonstrates steady state evaluation of these parameters. Figures 9 and 10 illustrate on-line detection of noise shifts in v, and simultaneous shifts in v and w, respectively.

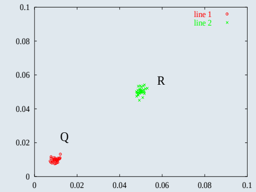

Figure 8 : Recovery of Q and R

$$Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} \quad R = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$$

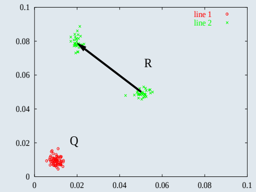

Figure 9 : Detection of R shift

$$Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} \quad R = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.02 & 0 \\ 0 & 0.08 \end{bmatrix}$$

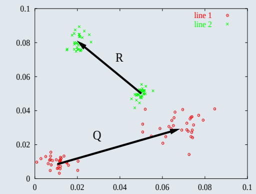

Figure 10 : Detection of Q and R shift

$$Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.07 & 0 \\ 0 & 0.03 \end{bmatrix} \quad R = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.02 & 0 \\ 0 & 0.08 \end{bmatrix}$$

7 Future work

Short term additions to project:

- Closed loop noise detection
- Semi-definite programming (SDP) solution
- Model mismatch

Long term additions to project:

- Non-Gaussian noise
- MPC control