Robust Offset Free Model Predictive Control

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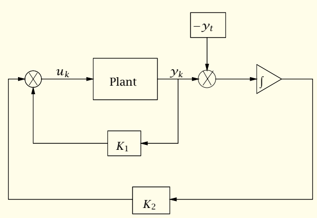
Research Objective

- Develop a new robust MPC theory that stabilizes a system with a polytopic model uncertainty description Ω. $(A, B) \in \Omega$ if

$$A = \sum_{i=1}^{I} \mu_i A_i, \quad B = \sum_{i=1}^{I} \mu_i B_i, \quad \sum_{i=1}^{I} \mu_i = 1, \quad \mu_i \geq 0$$

- Add a tree trajectory to the regulator to simulate time-varying uncertainties.
- Modify the controller formulation to include integral control offset free non-zero set point tracking.
- Evaluate the closed-loop stability.

Integral Control With ARMAX Models



Remarks:

- The process models are Auto Regressive Moving Average Exogenous Inputs (ARMAX) polytopic models.
- The state is defined as

$$x_k = [y_k^T, \ldots, y_{k-n-1}^T, u_{k-n-1}^T, \ldots, u_{k-1}^T \epsilon_k^T]^T$$

- Integral control is achieved by taking control action on ϵ_k.

$$\epsilon_k = \sum_{j=0}^{k} (y_t - y_j)$$

- K_1 and K_2 are the feedback gains on the extended state and ϵ_k, respectively.
- Offset free integral control is achieved when $\epsilon_k \longrightarrow 0 \quad \Longrightarrow \quad y_k \longrightarrow y_t$.
- Target calculation is unnecessary.

Infinite Horizon Regulator

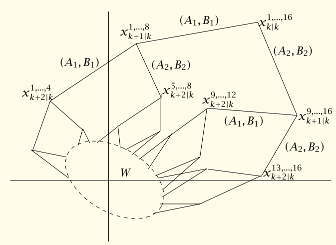


Figure 1: Regulator tree trajectory for $I = 2$ and $N = 4$.

$$\min_{\pi_k^l} \max_{l=1,\ldots,N_l} \Phi_k^l$$

subject to

$$x^l_{k+j+1|k} = A_{\upsilon_l(j+1)} x^l_{k+j|k} + B_{\upsilon_l(j+1)} u^l_{k+j|k} + z^l_{k+j|k}$$

$$x^l_{k+j|k} \in W \ \forall j \geq N, \quad x^l_{k+j|k} \in X, \quad u^l_{k+j|k} \in U$$

Remarks:

- Φ_k^l is the performance objective for trajectory l at time k.

$$\Phi_k^l = x^{lT}_{k+N|k} F x^l_{k+N|k} + \sum_{j=0}^{N-1} x^{lT}_{k+j|k} Q x^l_{k+j|k} + u^{lT}_{k+j|k} R u^l_{k+j|k}$$

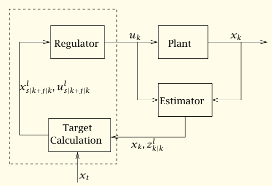
- W is the invariant terminal region in which the control policy $u^l_{k+j|k} = K x^l_{k+j|k}$ is robustly stabilizing.
- $F = F^T > 0$ is the final state penalty matrix.

$$F = \sum_{j=N}^{\infty} x^{lT}_{k+j|k} Q x^l_{k+j|k} + u^{lT}_{k+j|k} R u^l_{k+j|k}$$

  in which $Q \geq 0$ and $R > 0$.
- $\upsilon_l \in \mathbb{R}^N$ is the row vector containing the model indices necessary to simulate the time-varying uncertainties.
- N_l is the maximum number of possible trajectories, $N_l = I^N$.
- $(A, B) \in \Omega$ is robustly stabilizable if there exists K and F such that

$$F - Q - K^T R K - (A_i + B_i K)^T F (A_i + B_i K) \geq 0 \quad \forall i = 1, \ldots, I$$

Robust MPC With Target Calculation



$$\min_{u_s, x^l_{s|k+j|k}} \max_{l=1,\ldots,N_l} (x^l_{s|k+j|k} - x_t)^T Q_s (x^l_{s|k+j|k} - x_t)$$

subject to

$$x^l_{s|k+j|k} = A_{\upsilon_l(j)} x^l_{s|k+j|k} + B_{\upsilon_l(j)} u^l_{s|k+j|k} + z^l_{k+j|k}$$

$$u^l_{s|k+j|k} \in U \qquad x^l_{s|k+j|k} \in X$$

Remarks:

- Assume state is measurable.
- The integral model is $z^l_{k+j|k} \in \mathbb{R}^n$, the disturbance value at time j in the prediction horizon with state measurement up to time k.
- To simulate time-varying uncertainties, $z^l_{k+j|k}$ is not constant but time-varying for $j > 0$.
- The time-varying trajectories for $z^l_{k+j|k}$ in the regulator cause $x^l_{s|k+j|k} \in \mathbb{R}^n$ and $u^l_{s|k+j|k} \in \mathbb{R}^m$, the steady-state trajectories with measurement up to time k, to be time-varying as well.
- Robust stability is dependent on the regulator dynamics and the target calculation.
- Offset free integral control for non-zero set point tracking can be accomplished if there exist $\bar{K}$ and $\bar{F}$ such that

$$\bar{F} - \bar{Q} - \bar{K}^T \bar{R} \bar{K} - (\bar{A}_i + \bar{B}_i \bar{K})^T \bar{F} (\bar{A}_i + \bar{B}_i \bar{K}) \geq 0 \quad i = 1, \ldots, I$$

  in which

$$\begin{bmatrix} u^l_{s|k+j+1|k} \\ x^l_{k+j+1|k} \end{bmatrix} = \bar{A}_{\upsilon_l(j+1)} \begin{bmatrix} u^l_{s|k+j|k} \\ x^l_{k+j|k} \end{bmatrix} + \bar{B}_{\upsilon_l(j+1)} \begin{bmatrix} u^l_{s|k+j|k} \\ u^l_{k+j|k} \end{bmatrix}$$

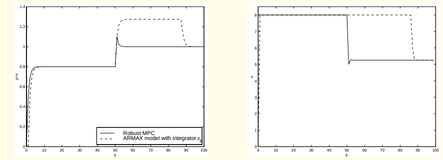
$$\bar{Q} = \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix} \quad \bar{R} = \begin{bmatrix} R & -R \\ -R & R \end{bmatrix} \quad \bar{K} = \begin{bmatrix} I & 0 \\ I & K \end{bmatrix} \quad \bar{F} = \begin{bmatrix} I & 0 \\ 0 & F \end{bmatrix}$$

- An LMI-based optimizer is used to solve for $\bar{K}$ and $\bar{F}$ subject to the equality constraints.

Constraint Saturation Example

$$G_1 = \frac{0.1}{s+10} \qquad G_2 = \frac{1}{s+10}$$

$$y(s) = G_i u(s)$$
$$-\infty \leq u_k \leq 8$$
$$-\infty \leq y_k \leq \infty$$



Remarks:

- Model 2 is the plant and $y_t = 1$.
- The input constraint causes the set point to be unreachable.
- A disturbance at time $k = 50$ enters the system. The set point is reachable.
- Both robust MPC with target calculation and integral control with ARMAX models successfully reject the disturbance and reach the set point.
- But integral control with the ARMAX model exhibits windup behavior by delaying decreasing u_k from its constraint.

Conclusion and Future Work

Conclusion

- Developed robust MPC theory that controls a time-varying uncertain system described by $(A, B) \in \Omega$.
- Modified the theory to include integral control that did not exhibit wind-up.
- Developed stability conditions that guaranteed offset free performance achieved without windup.

Future Work

- Replace state measurement with output measurement and state estimation.
- Study the model uncertainty description to see if it is possible to characterize the set $(A, B) \in \Omega$ for which robust offset free control is possible.