Disturbance modeling for robust control of ill-conditioned processes with MPC

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Introduction

A multivariable process is ill-conditioned when some manipulated inputs have almost the same effect on the controlled outputs. Ill-conditioned processes are frequent in the process industries (*e.g.* distillation processes). They show large interactions and directionality.

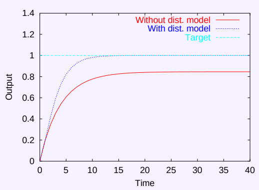
If an ill-conditioned process is controlled with decentralized loops (*e.g.* PI), the performance suffers from interactions between control loops. On the other hand, if we use DMC controllers (centralized), excellent nominal performance is obtained, but poor robustness to input uncertainty is shown.

MPC and disturbance models

Necessity of a disturbance model

Example:

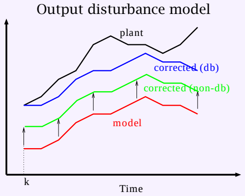
$$g_m(s) = \frac{1}{10s+1}$$
$$g_p(s) = \frac{1.5}{10s+1}$$



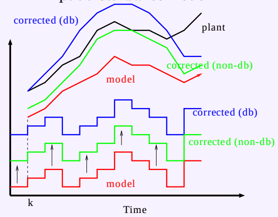
A disturbance model is required to correct the model forecasting and to obtain offset-free control in the presence of plant-model mismatch and/or unmodeled disturbances.

Input and output disturbance model

Output disturbance model



Input disturbance model



- They guarantee offset-free control (Muske and Rawlings, 1993)
- Pure output disturbances are unlikely to occur in the process industries. *In fact, the load always enters upstream of a dominant time constant and, in many cases, at the same point as the manipulated variable* (Shinskey, 1994)
- DMC, which uses an output disturbance model, does not quickly reject slow disturbances (Morari and Lee, 1991)

Optimal disturbance model

We define a general disturbance model as:

$$x_{k+1} = Ax_k + Bu_k + Dd_k + \xi_k^x$$
$$d_{k+1} = d_k + \xi_k^d$$
$$y_k = Cx_k + Pd_k + v_k$$

A state-space representation of the closed-loop system is:

$$z_{k+1} = \Lambda z_k + \Xi \tilde{y} + \Theta \tilde{d}$$
$$y_k = \Gamma z_k + P_p \tilde{d}$$

in which $z_k = \begin{bmatrix} x_k & \hat{x}_{k|k} & \hat{d}_{k|k} & u_{k-1}\end{bmatrix}^T$. The closed-loop true objective function is found by solving a Lyapunov equation:

$$\Phi = \tilde{z}_0^T \tilde{S} \tilde{z}_0$$
$$\tilde{S} = \tilde{Q} + \Psi^T \tilde{S} \Psi$$

in which $\tilde{z}_0 = \begin{bmatrix} z_0^T & \tilde{y}^T & \tilde{d}^T\end{bmatrix}^T$.

We find the "optimal" disturbance model, which guarantees the best performance in the worst case of plant uncertainty:

$$\min_{D,P}\left[\max_{\text{Plant uncertainty}} \Phi\right]$$

Case study: composition control of a distillation column

Model and uncertainty

Skogestad and Morari (1987) with minor changes (Lundström *et al.* , 1995):

$$\begin{bmatrix} y_D \\ x_B\end{bmatrix} = e^{-\theta s}\begin{bmatrix} \frac{0.878}{1+\tau_1 s} & \left(\frac{0.014}{1+\tau_2 s} - \frac{0.878}{1+\tau_1 s}\right) \\ \frac{1.082}{1+\tau_1 s} g_L(s) & \left(\frac{-0.014}{1+\tau_2 s} - \frac{1.082}{1+\tau_1 s}\right)\end{bmatrix}\begin{bmatrix} L_t \\ V_b\end{bmatrix}$$

$$\tau_1 = 194; \quad \tau_2 = 15; \quad \theta = 1$$

$$g_L(s) = \frac{1}{(1+\theta_L/5s)^5}$$

Indepedent input uncertainty is considered:

$$L_t^{\text{actual}} = L_t^{\text{computed}}(1+\delta_1)$$
$$V_b^{\text{actual}} = V_b^{\text{computed}}(1+\delta_2)$$

in which $|\delta_1, \delta_2| \leq 0.2$

Optimal disturbance model

The general disturbance model is:

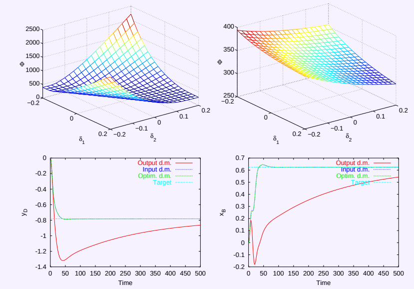
$$D = B\begin{bmatrix} \omega_1^D & 0 \\ 0 & \omega_2^D\end{bmatrix}; \qquad P = \begin{bmatrix} \omega_1^P & 0 \\ 0 & \omega_2^P\end{bmatrix}$$

in which $\omega_i^D, \omega_i^P \in [0,1]$. For several cases of setpoint change and disturbance rejection, the optimal disturbance model is found by solving:

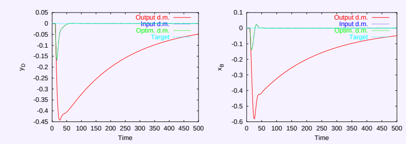
$$\min_{\omega_1^D,\omega_2^D,\omega_1^P,\omega_2^P} \max_{\delta_1,\delta_2} \Phi$$

Setpoint change in the unfavorable direction

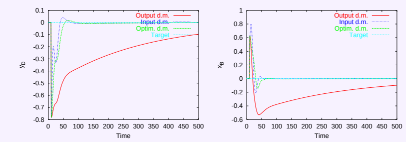
Output disturbance model — Input disturbance model



Rejection of an input disturbance in the favorable direction



Rejection of an output disturbance in the unfavorable direction



Comments

- Repeating the optimization for different directions of the setpoint and of the plant disturbance we found that the optimal disturbance models change slightly
- For setpoint changes and input disturbance rejections, the optimal disturbance models are close to the input disturbance model
- For output disturbance rejections the optimal disturbance models are a combination of input and output disturbance model

Conclusions

- Analyzed the implications of the disturbance model on the robustness of MPC for ill-conditioned processes
- Proposed a method for searching the "optimal" disturbance model by solving a min-max optimization problem
- For ill-conditioned processes, the output disturbance model as in DMC shows poor robustness to input uncertainty
- For ill-conditioned processes, the input disturbance model is more robust and, in several cases, is close to the optimal disturbance model