Disturbance modeling for robust control of ill-conditioned processes with MPC

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Outline

- Ill-conditioned processes:
  - an overview
  - what's the deal?
  - literature review
- Disturbance modeling:
  - MPC, necessity of a disturbance model
  - input and output disturbance model
  - "optimal" disturbance model
- A case study
- Conclusions

Ill-conditioned processes:
an overview

- A multivariable process is ill-conditioned when some manipulated inputs have almost the same effect on the controlled outputs

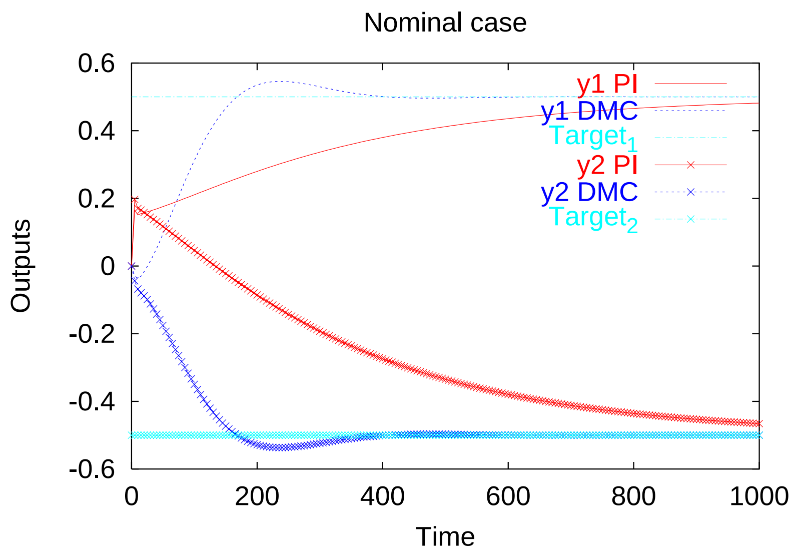
- Ill-conditioned processes are frequent in the process industries (*e.g.* distillation processes)

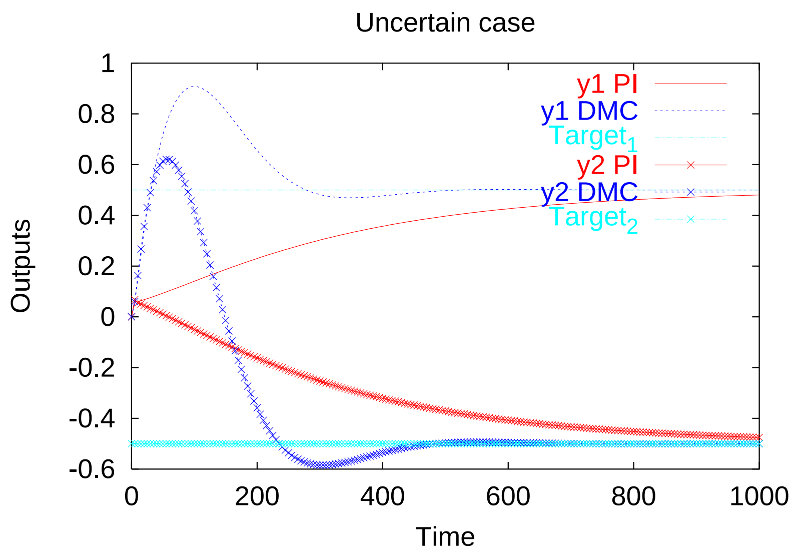
- Large interactions between control loops

- Directionality:
  - some setpoint changes (or disturbances) can be obtained (or rejected) with small changes of the inputs, while other setpoint changes (or disturbances) require great changes of the inputs

Ill-conditioned processes: what's the deal?

(Morari and Zafiriou, 1989)





Ill-conditioned process: literature review

- Feedback controllers

  1. Decentralized controllers: good robustness to model errors but poor performance due to strong interactions (Skogestad and Morari, 1987)
  2. Inverse-based controllers based on SVD (Brambilla and D'Elia, 1992)

- IMC controllers: double filter structure (Morari, and Zafiriou, 1989), (Semino *et al.* , 1993)

- Predictive controllers (DMC)

  1. Double integrated disturbances (Lundström *et al.* , 1995)
  2. Robust modified models (Pannocchia and Semino, 1999)

MPC: necessity of a disturbance model

- Model Predictive Control is a control technique in which a model is used to forecast the future behavior of the process, and the input sequence is computed in order to minimize a performance objective function over a (finite) prediction horizon

- Usually, only the first input is injected into the plant (receding horizon control)

- Models are not perfect forecasters of the process behavior: steady-state offset is obtained in the presence of plant-model mismatch and/or unmodeled disturbances

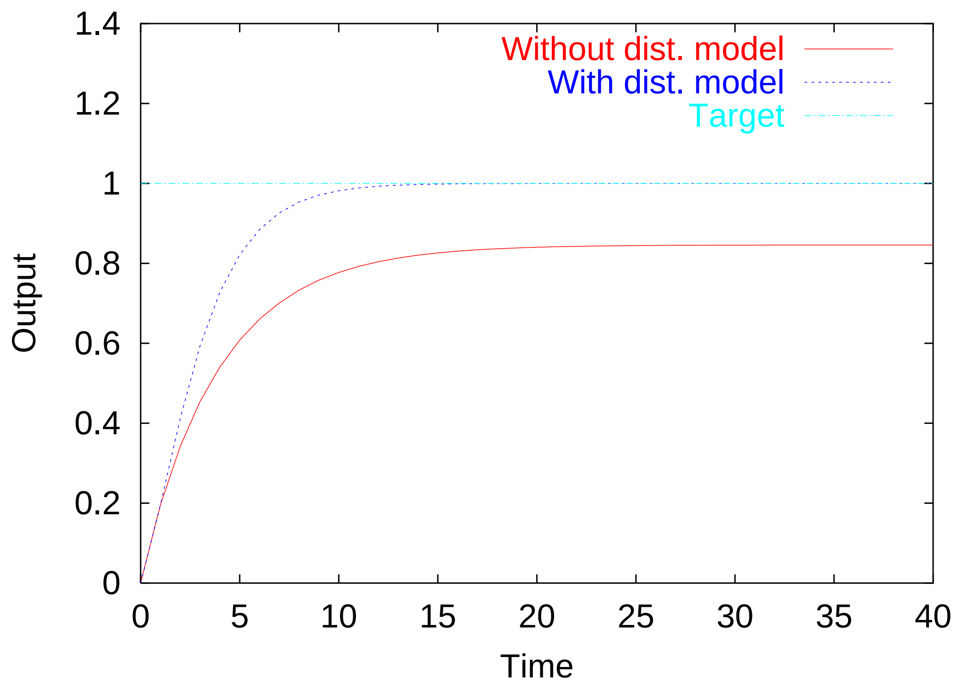
- Disturbance model: correct the model forecast and obtain offset-free control

MPC: necessity of a disturbance model (cont'd)

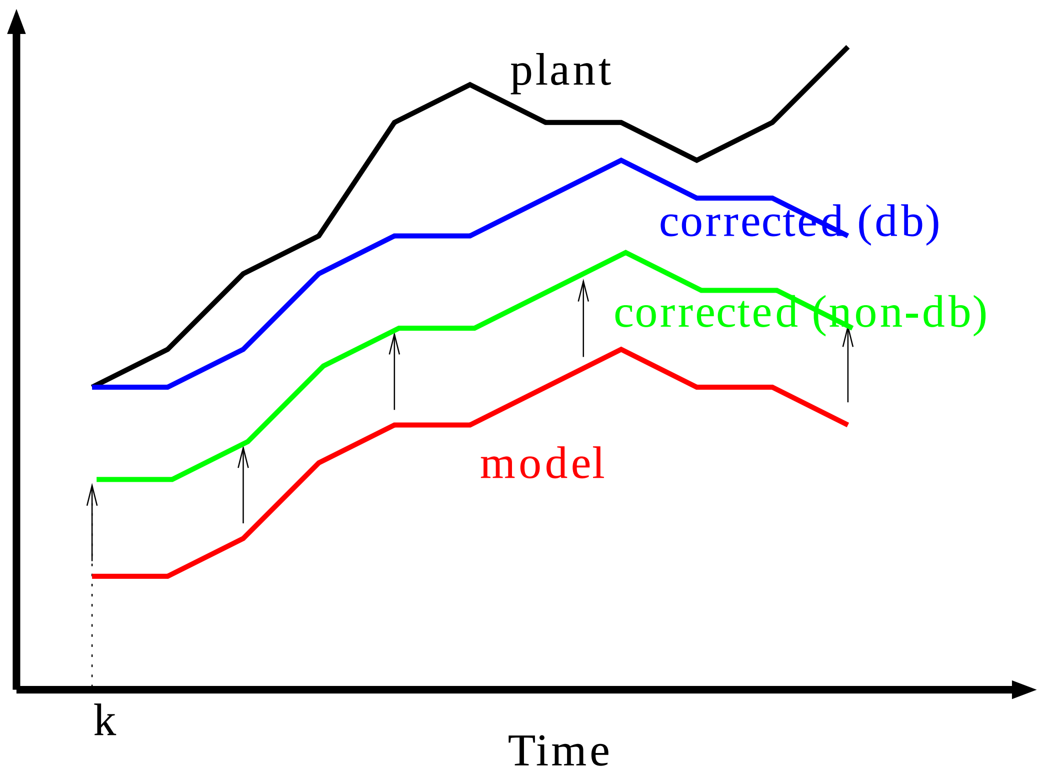
Example:

$$g_m(s) = \frac{1}{10s+1}$$

$$g_p(s) = \frac{1.5}{10s+1}$$



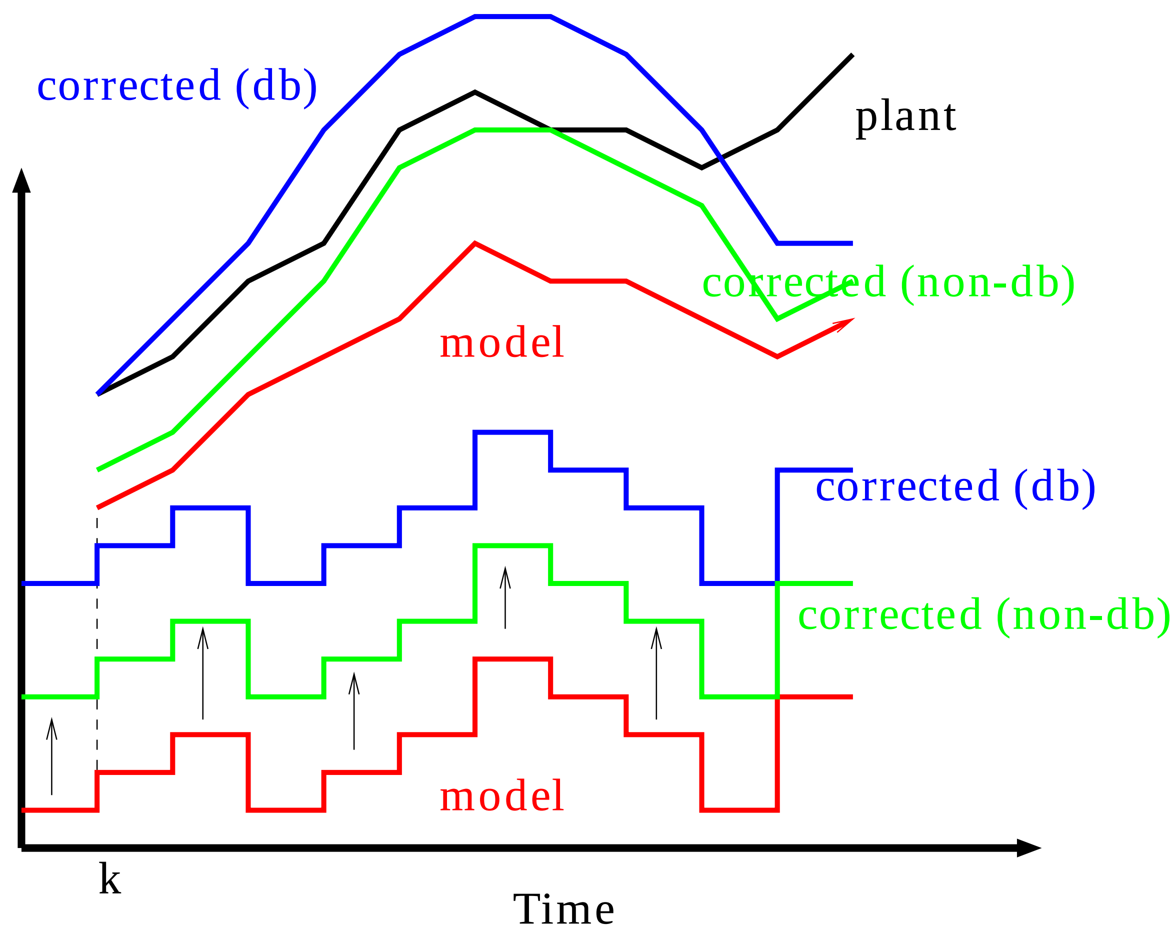
Output disturbance model



- The difference between the actual and the predicted output is assumed to be caused by a step disturbance added at the process output, which remains constant in the future

- It is used by the industrial implementations of MPC

Input disturbance model



- The difference between the actual and the predicted output is assumed to be caused by a step disturbance added at the process input, which remains constant in the future

Mathematical formulation

- Output disturbance model:

$$
\begin{aligned}
x_{k+1} &= Ax_k + Bu_k + \xi_k^x \\
d_{k+1} &= d_k + \xi_k^d \\
y_k &= Cx_k + d_k + v_k
\end{aligned}
$$

- Input disturbance model:

$$
\begin{aligned}
x_{k+1} &= Ax_k + B(u_k + d_k) + \xi_k^x \\
d_{k+1} &= d_k + \xi_k^d \\
y_k &= Cx_k + v_k
\end{aligned}
$$

- Dead-beat tuning:

$$
R_v \to 0; \qquad \frac{Q_x}{Q_d} \to 0
$$

Output *vs* input disturbance model

- They guarantee offset-free control (Muske and Rawlings, 1993)

- Pure output disturbances are unlikely to occur in the process industries. *In fact, the load always enters upstream of a dominant time constant and, in many cases, at the same point as the manipulated variable* (Shinskey, 1994)

- DMC, which uses an output disturbance model, does not quickly reject slow disturbances (Morari and Lee, 1991)

- In general, the input disturbance model leads to a more aggressive control action

“Optimal” disturbance model

- Combine input and output disturbance model

$$x_{k+1} = Ax_k + Bu_k + Dd_k + \xi_k^x$$
$$d_{k+1} = d_k + \xi_k^d$$
$$y_k = Cx_k + Pd_k + v_k$$

- State estimation using the Kalman filter:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + L_x(y_k - C\hat{x}_{k|k-1} - P\hat{d}_{k|k-1})$$
$$\hat{d}_{k|k} = \hat{d}_{k|k-1} + L_d(y_k - C\hat{x}_{k|k-1} - P\hat{d}_{k|k-1})$$

- Closed-loop evolution:

$$z_{k+1} = \Lambda z_k + \Xi \bar{y} + \Theta \bar{d}$$
$$y_k = \Gamma z_k + P_p \bar{d}$$

in which $z_k = \begin{bmatrix} x_k & \hat{x}_{k|k} & \hat{d}_{k|k} & u_{k-1} \end{bmatrix}^T$

"Optimal" disturbance model (cont'd)

- Evaluation of the closed-loop true objective function by solving a Lyapunov equation:

$$\Phi = \tilde{z}_0^T \tilde{S} \tilde{z}_0$$
$$\tilde{S} = \tilde{Q} + \Psi^T \tilde{S} \Psi$$

in which $\tilde{z}_0 = \begin{bmatrix} z_0^T & \bar{y}^T & \bar{d}^T \end{bmatrix}^T$

- Find the "optimal" disturbance model, which guarantees the best performance in the worst case of plant uncertainty
  - min-max problem
  - the solution depends on the size of the uncertain plant region, on the setpoint reference and on the plant disturbance
  - square systems, and constraints never active

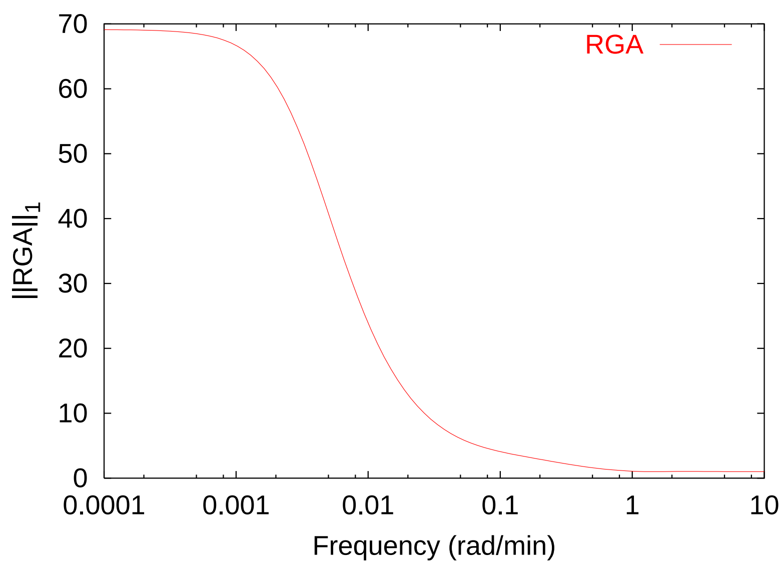
Composition control of a distillation column

The model (Skogestad and Morari, 1987) with minor changes (Lundström *et al.* , 1995):

$$\begin{bmatrix} y_D \\ x_B \end{bmatrix} = e^{-\theta s} \begin{bmatrix} \frac{0.878}{1+\tau_1 s} & \left(\frac{0.014}{1+\tau_2 s} - \frac{0.878}{1+\tau_1 s} \right) \\ \frac{1.082}{1+\tau_1 s} g_L(s) & \left(\frac{-0.014}{1+\tau_2 s} - \frac{1.082}{1+\tau_1 s} \right) \end{bmatrix} \begin{bmatrix} L_t \\ V_b \end{bmatrix}$$

$$\tau_1 = 194; \quad \tau_2 = 15; \quad \theta = 1$$

$$g_L(s) = \frac{1}{(1 + \theta_L/5s)^5}$$



Plant uncertainty and min-max optimization

- Independent input uncertainty:

$$L_t^{\text{actual}} = L_t^{\text{computed}}(1 + \delta_1)$$
$$V_b^{\text{actual}} = V_b^{\text{computed}}(1 + \delta_2)$$

in which $|\delta_1, \delta_2| \leq 0.2$

- General disturbance model:

$$D = B \begin{bmatrix} \omega_1^D & 0 \\ 0 & \omega_2^D \end{bmatrix}; \qquad P = \begin{bmatrix} \omega_1^P & 0 \\ 0 & \omega_2^P \end{bmatrix}$$

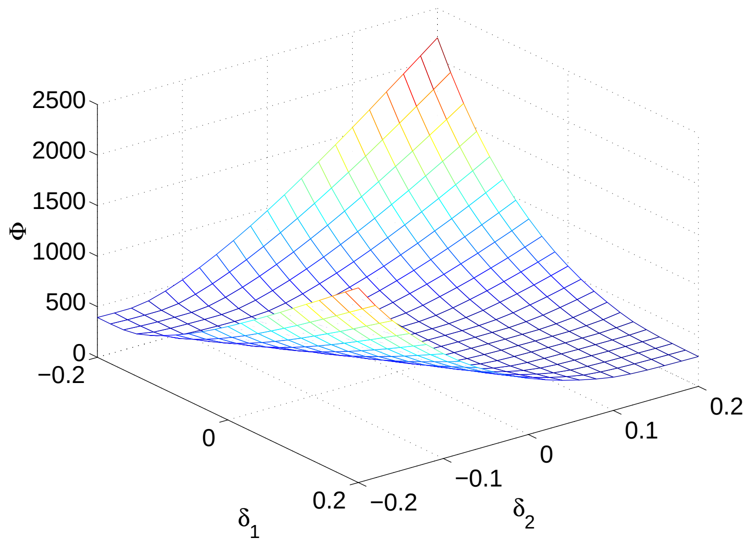
in which $\omega_i^D, \omega_i^P \in [0, 1]$

- Min-max optimization:

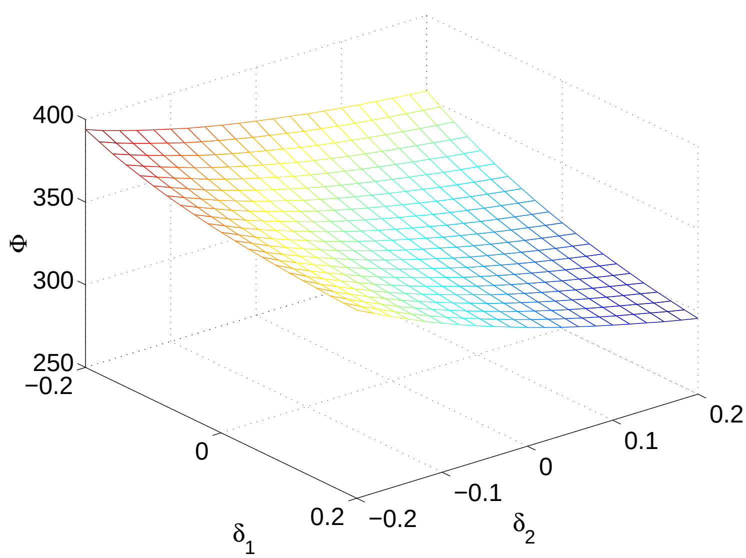
$$\min_{\omega_1^D, \omega_2^D, \omega_1^P, \omega_2^P} \max_{\delta_1, \delta_2} \Phi$$

Setpoint change in the unfavorable direction

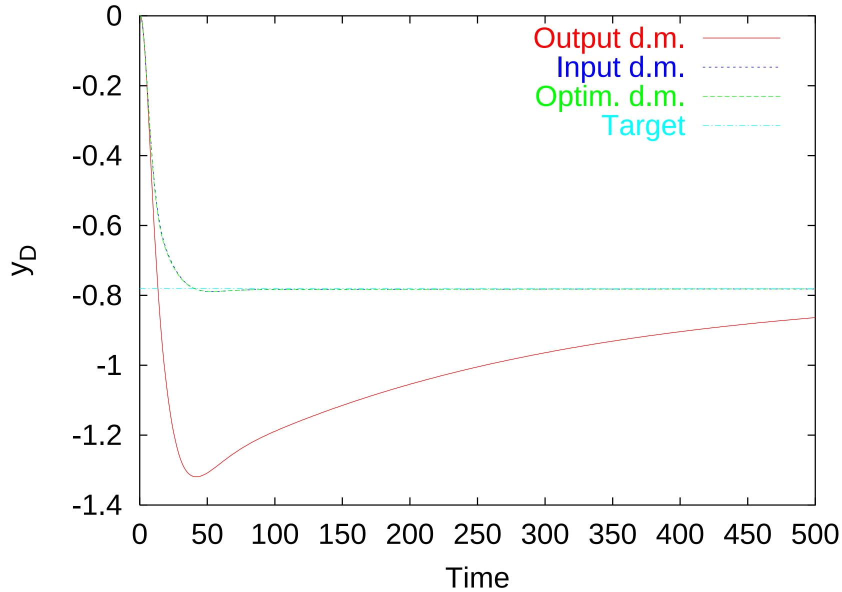
Output disturbance model

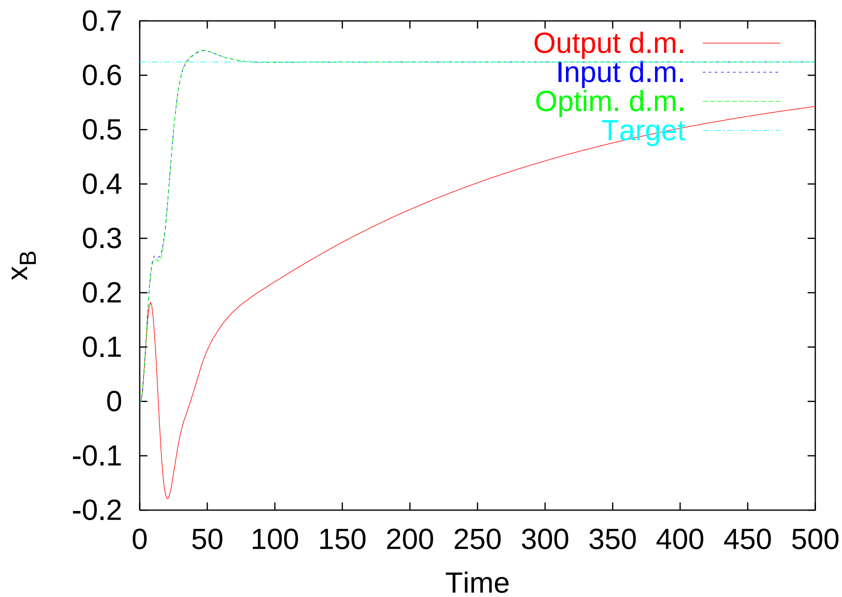


Input disturbance model

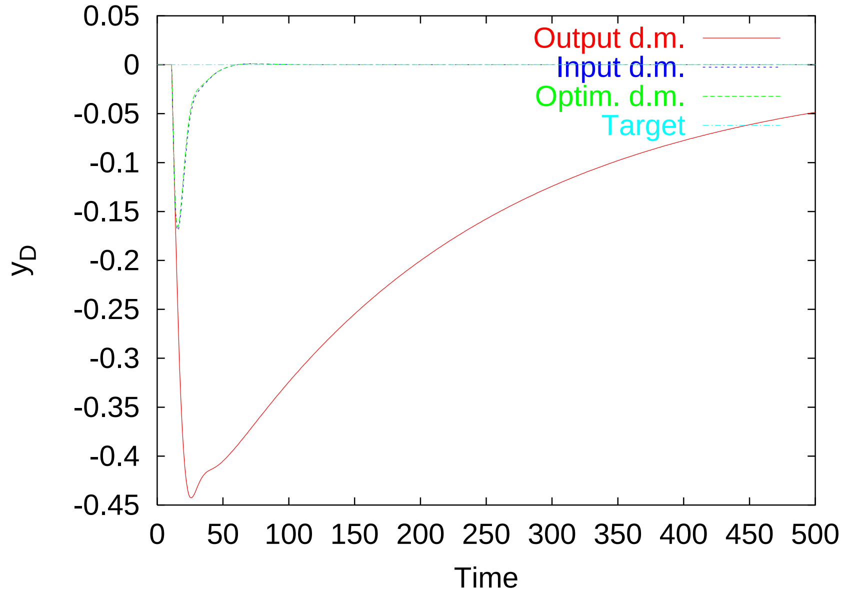


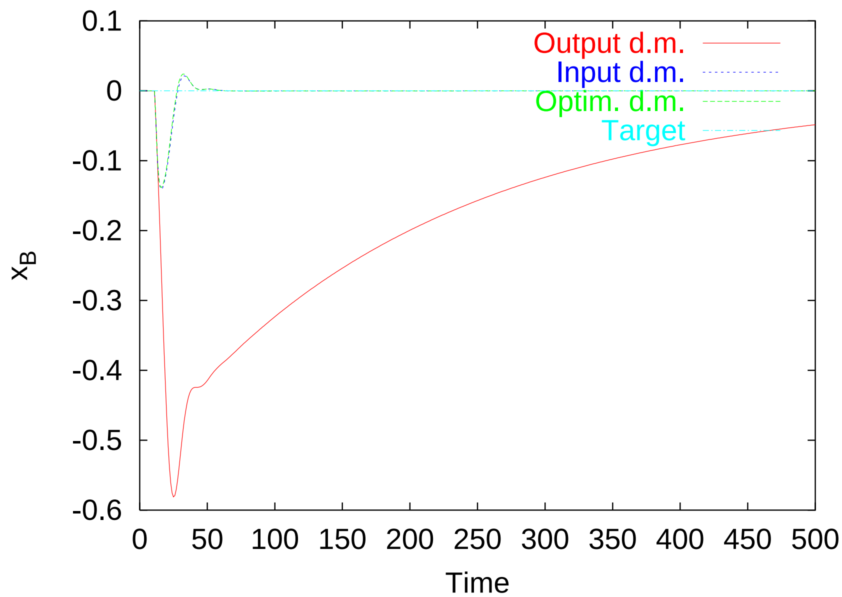
Setpoint change in the unfavorable direction (cont'd)



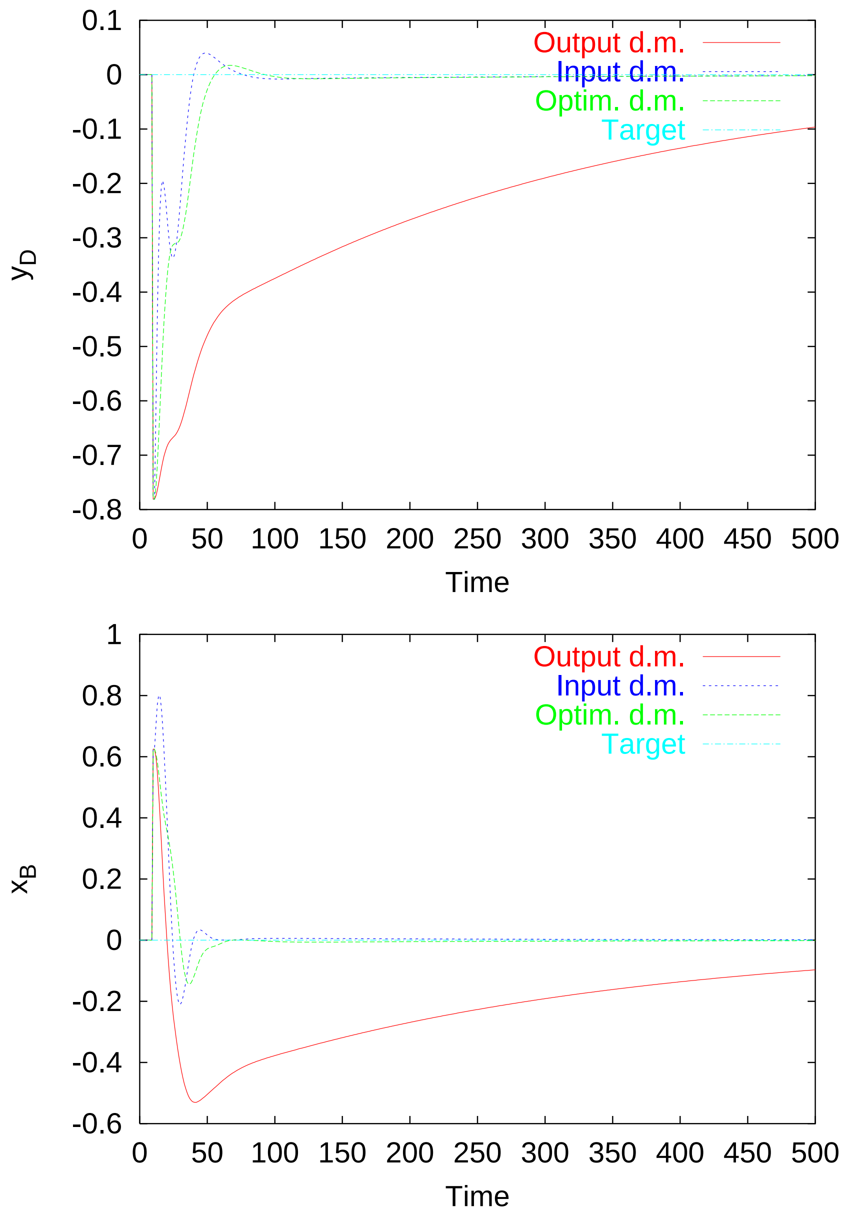


Rejection of an input disturbance in the favorable direction





Rejection of an output disturbance in the unfavorable direction



Comments

- Repeating the optimization for different directions of the setpoint and of the plant disturbance we found that the optimal disturbance models change slightly
- For setpoint changes and input disturbance rejections, the optimal disturbance models are close to the input disturbance model
- For output disturbance rejections the optimal disturbance models are a combination of input and output disturbance model

Conclusions

- Analyzed the implications of the disturbance model on the robustness of MPC for ill-conditioned processes

- Proposed a method for searching the “optimal” disturbance model by solving a min-max optimization problem

- For ill-conditioned processes, the output disturbance model as in DMC shows poor robustness to input uncertainty

- For ill-conditioned processes, the input disturbance model is more robust and, in several cases, is close to the optimal disturbance model

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