Model Predictive Control and Identification – An Adaptive Control Paradigm

TWMCC

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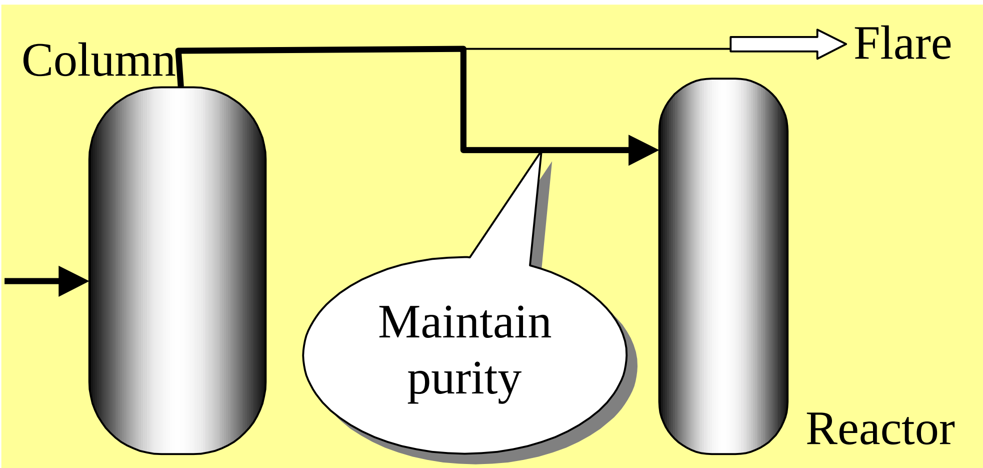
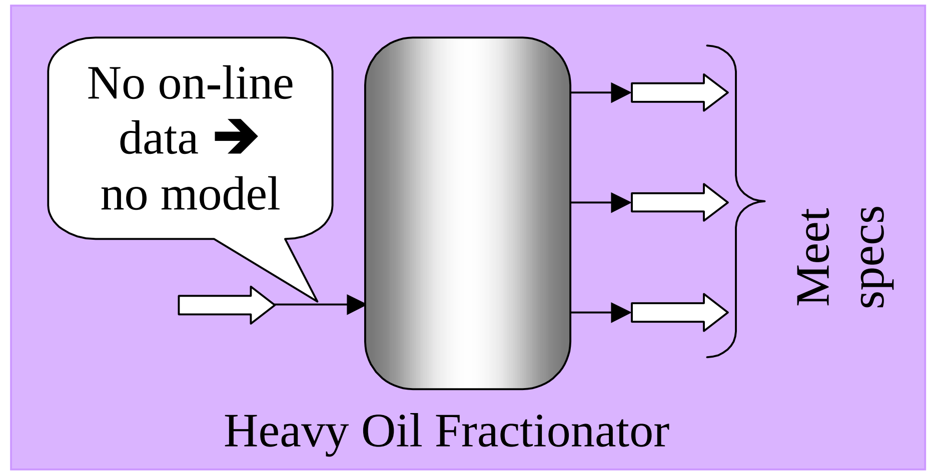
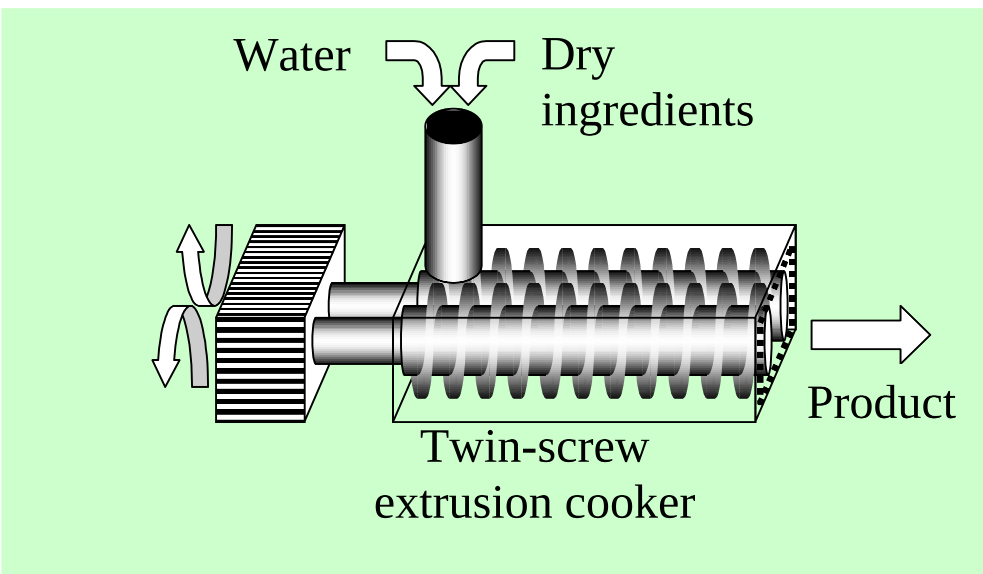
Outline

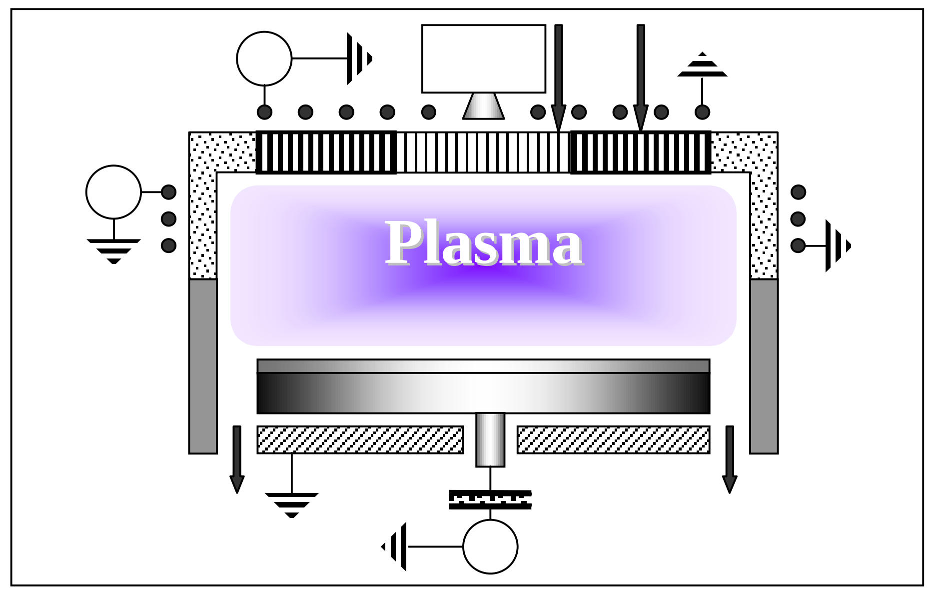
- Introduction
- Motivating examples
- Three approaches to closed-loop identification and adaptive control
- Conclusions

The physical picture





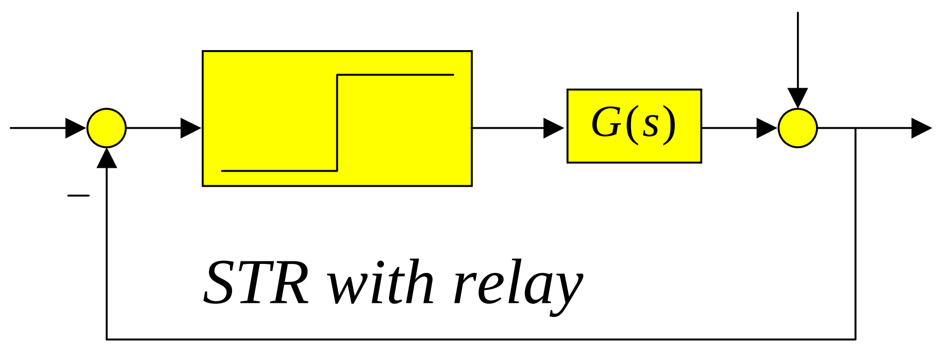




Motivating Example

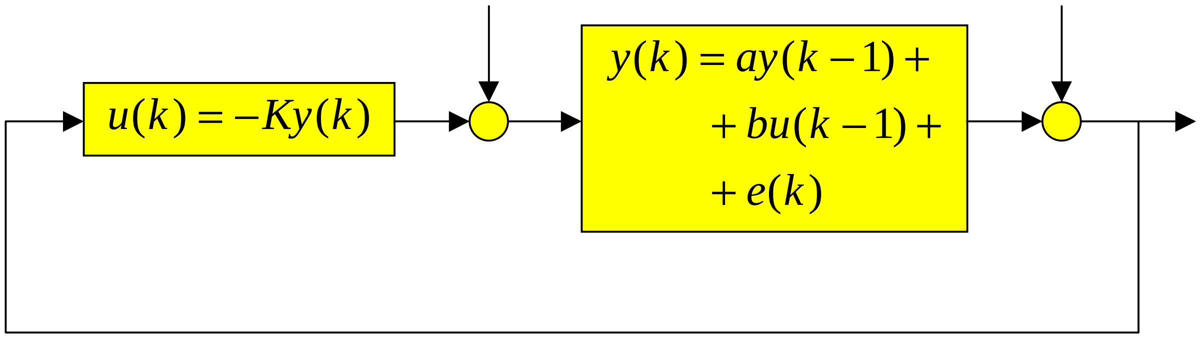
"*Closed-loop identification is impossible without external excitation*"

But...



Motivating Example

Estimate a, b



$$y(k) = (a - bK)y(k-1) + e(k)$$

$$\Downarrow$$

$$\hat{a} = a + \gamma K$$
$$\hat{b} = b + \gamma$$

$$\Downarrow$$

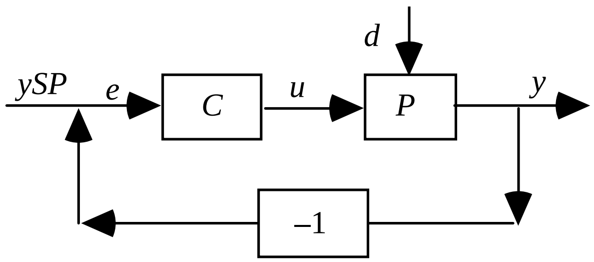
"*Identification impossible*"

But Ziegler-Nichols works!

???

Closed-loop Identification and Adaptive Control – Revisited

Closed-loop identification – a unifying view



$$\min_{P_m} \|P_m u - y\|$$

$$u = Ce \overset{y^{SP}=0}{=} C(-y) \Rightarrow -y = C^{-1}u$$

$$\Downarrow$$

$$\min_{P_m} \|P_m u - y\| = \min_{P_m} \|P_m u + C^{-1}u\| = \min_{P_m} \|(P_m + C^{-1})u\|$$

$$\Downarrow$$

$$\tilde{P} = -C^{-1}$$

Remedies

- C^{-1} does not exist (STR with relay)
- $\tilde{P}$, C^{-1} belong to different classes of mappings (Ziegler-Nichols tuning, adaptation, causal $\tilde{P}$/noncausal C^{-1})
- $y^{SP} \neq 0$

Motivating Example – Revisited

$$\min_{\hat{a},\hat{b}} E\left[\sum_{i=0}^{k}\left(\hat{y}(i|i-1)-y(i)\right)^2\right]=$$

$$\min_{\hat{a},\hat{b}} E\sum_{i=0}^{k}\left(\hat{a}y(i-1)+\hat{b}u(i-1)+e(i)-y(i)\right)^2$$

$$\Downarrow$$

$$\begin{bmatrix}\hat{a}\\ \hat{b}\end{bmatrix}=(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

But

$$\mathbf{X}=\begin{bmatrix} y(0) & u(0)\\ \vdots & \vdots \\ y(k-1) & u(k-1)\end{bmatrix}$$

does not have full rank!

Motivating Example – Revisited (Cont'd)

$$y(k) = \sum_{i=1}^{m} b_i u(k-i) + e(k)$$

$$\Downarrow$$

$$\min_{\hat{b}_j} E \sum_{i=0}^{k} \left(\sum_{j=1}^{m} \hat{b}_j u(i-j) + e(i) - y(i) \right)^2$$

$$\begin{bmatrix} \hat{b}_1 \\ \vdots \\ \hat{b}_m \end{bmatrix} = (\mathbf{U}^T \mathbf{U})^{-1} \mathbf{U}^T \mathbf{y}$$

$$\mathbf{U} \triangleq \begin{bmatrix} u(-1) & \cdots & u(-m) \\ \vdots & \ddots & \vdots \\ u(k-1) & \cdots & u(k-m) \end{bmatrix}$$ **has full rank!**

Closed-loop Identification and Adaptive Control – Revisited

$$\min_{\hat{a},\hat{b}} E\left[\sum_{i=0}^{k}\left(\hat{y}(i|i-1)-y(i)\right)^2\right]=$$

$$\min_{\hat{a},\hat{b}} E\sum_{i=0}^{k}\left(\hat{a}y(i-1)+\hat{b}u(i-1)+e(i)-y(i)\right)^2$$

$$\Downarrow$$

$$\begin{bmatrix}\hat{a}\\ \hat{b}\end{bmatrix}=(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

Require

(a) $\mathbf{X}^T\mathbf{X}$ invertible and well conditioned

(b) u, ε uncorrelated

$$\mathbf{X}^T\mathbf{X}\succeq\rho\mathbf{I}$$

$$\Downarrow$$

MPCI

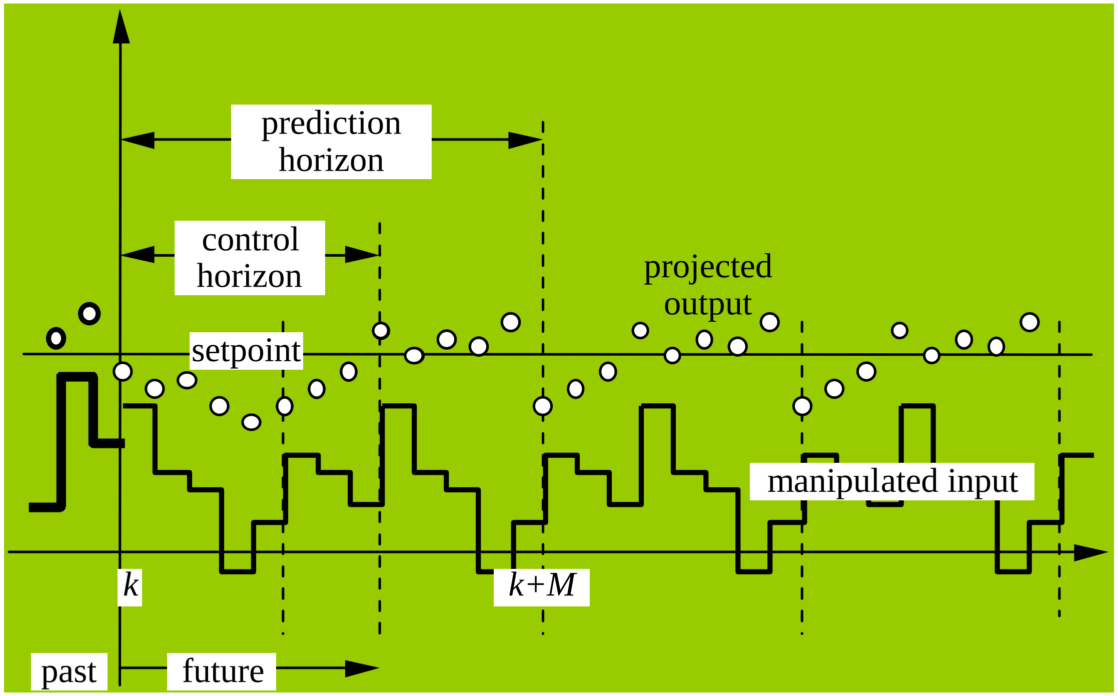
MPCI: Expanding the MPC paradigm to adaptive control

$$\underset{\substack{\textit{process input values}\\ \textit{over control horizon}}}{\text{minimize}} \begin{bmatrix} \textit{control objective over} \\ \textit{optimization horizon} \end{bmatrix}$$

subject to

Standard MPC constraints

Persistent excitation constraints over finite horizon.



MPCI quantitatively

Objective:

$$\min_{\substack{u(k+i-1|k)\\ \mu,\varepsilon,\rho}} \sum_{i=1}^{M}\left[\omega_i\left(y(k+i|k)-y^{sp}\right)^2 + r_i\Delta u(k+i-1|k)^2\right] + q_1\mu^2 + q_2\varepsilon^2 - q_3\rho$$

Input constraints:

$$u_{\max} \geq u(k+i-1|k) \geq u_{\min}, \quad i=1,2,\cdots,M$$

$$\Delta u_{\max} \geq \Delta u(k+i-1|k) \geq \Delta u_{\min}, \quad i=1,2,\cdots,M$$

Output constraints:

$$y_{\max} + \varepsilon \geq y(k+i|k) \geq y_{\min} - \varepsilon, \quad i=1,2,\cdots,M$$

$$y(k+i|k) = \phi(k+i-1)^T\,\overline{\theta}(k), \quad i=1,2,\cdots,M$$

$$\overline{\theta}(k) = \underbrace{\left[\sum_{j=1}^{s'}\phi(k-j)\phi(k-j)^T\right]}_{\text{Information matrix}}{}^{-1}\left[\phi(k-1) \quad \cdots \quad \phi(k-s')\right]\mathbf{y}(k)$$

$$\mathbf{y}(k) = [y(k) \cdots y(k-s+1)]^T$$

$$\phi(k-j-1)^T = [u(k-j-1)\cdots u(k-j-m)\,y(k-j-1)\cdots y(k-j-n)\,1]$$

PE constraints:

$$\sum_{j=1}^{s}\phi(k+i-j)\phi(k+i-j)^T \succeq (\rho-\mu)\mathbf{I}, \quad i=1,\cdots,M$$

$$\rho > \mu \geq 0$$

Key for Closed-Loop Identification

If no external dither,
Controller cannot be linear, time-invariant!

Controller is LTI $\Rightarrow$
Transfer-function analysis of convergence

Controller is *not* LTI $\Rightarrow$
Lyapunov analysis of convergence

MPCI variants

Weak PE of order N

$$u(t) = u_0 + \sum_{i=1}^{N/2} c_i \sin(\omega_i t + \varphi_i)$$

$$|c_i| \geq \sigma > 0$$

MPCI with Chance Constraints

$$P[y_{\max} + \varepsilon \geq y(k+i|k) \geq y_{\min} - \varepsilon] \geq 1 - \alpha$$

Multiscale MPCI

$$\min_{\substack{u(k+i|k) \\ \mu,\varepsilon,\rho}} \left(\mathbf{y}(k+M) - \mathbf{y}^{SP}\right)^T \Omega \left(\mathbf{y}(k+M) - \mathbf{y}^{SP}\right) + \ldots =$$

$$\min_{\substack{u(k+i|k) \\ \mu,\varepsilon,\rho}} \left(\tilde{\mathbf{y}}(k+M) - \tilde{\mathbf{y}}^{SP}\right)^T \mathbf{W}^T \Omega \mathbf{W} \left(\tilde{\mathbf{y}}(k+M) - \tilde{\mathbf{y}}^{SP}\right) + \ldots$$

MPCI variants (cont'd)

Variations of Least-Squares Identification

- Weighted Least-Squares and use of prior knowledge.
- Least-Squares with exponential data weighting.
- Least-Squares with covariance resetting.
- Least-Squares with covariance modification.

MPCI for various classes of process models

MPCI alternatives

Why?

- Simplicity
- Backward compatibility

MPC with exciting setpoint for process input

$$\min_{u(k+i-1|k)} \sum_{i=1}^{M} \left[\begin{array}{l} \left(y(k+i|k) - y^{sp} \right)^2 + \\ r_i \left(u(k+i-1|k) - u^{SP}(k+i-1) \right)^2 + \ldots \end{array} \right]$$

- Convenient for DMC structure
- Use in pro-active process monitoring

Subspace Identification

$$Y = \begin{bmatrix} y_1 & y_2 & \cdots & y_N \\ y_2 & y_3 & \cdots & y_{N+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_\alpha & y_{\alpha+1} & \cdots & y_{N+\alpha+1} \end{bmatrix}$$

$$U = \begin{bmatrix} u_1 & u_2 & \cdots & u_N \\ u_2 & u_3 & \cdots & u_{N+1} \\ \vdots & \vdots & \ddots & \vdots \\ u_\alpha & u_{\alpha+1} & \cdots & u_{N+\alpha+1} \end{bmatrix}$$

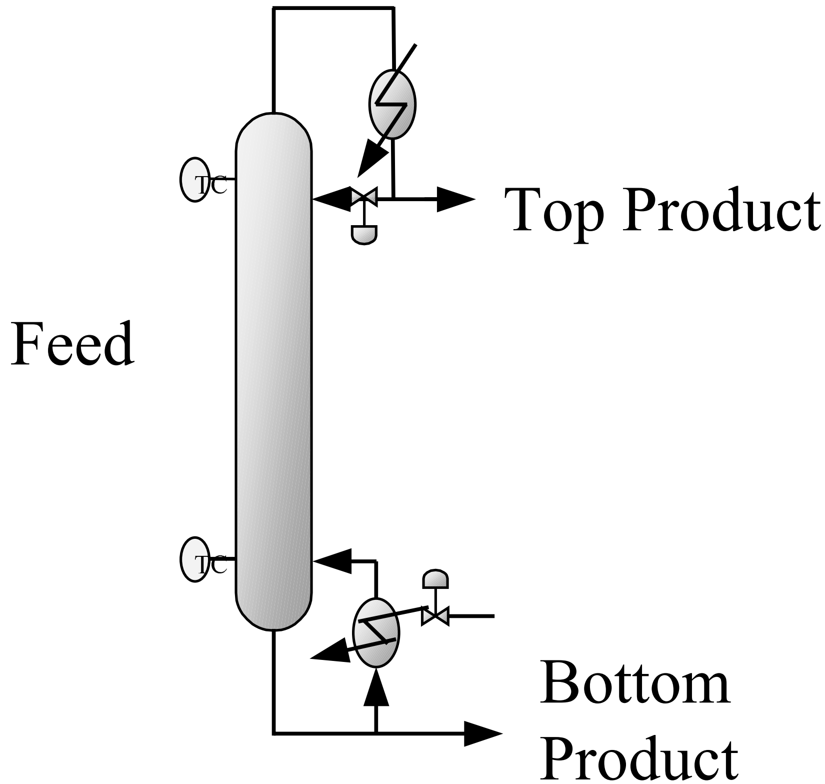
$$\Pi_{U^T}^{\perp} = I - U^T (UU^T)^{-1} U$$

$$Y \Pi_{U^T}^{\perp}$$

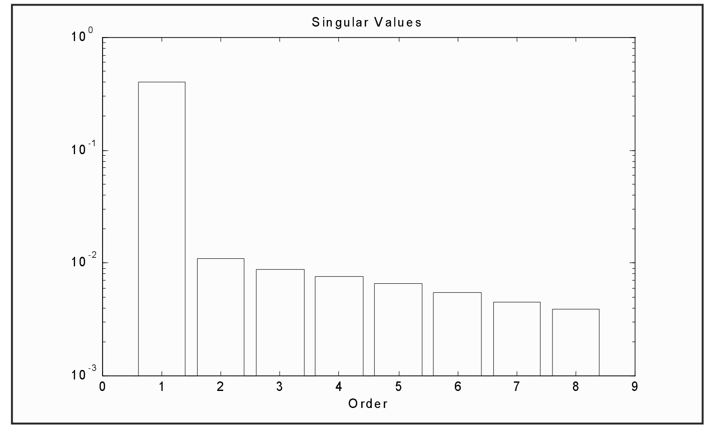
$$\text{SVD} \xrightarrow[\textit{n-order}]{} \Gamma \longrightarrow \text{A,B,C \& D}$$

Other Methods - IVM, Weightings

MOTIVATING EXAMPLE

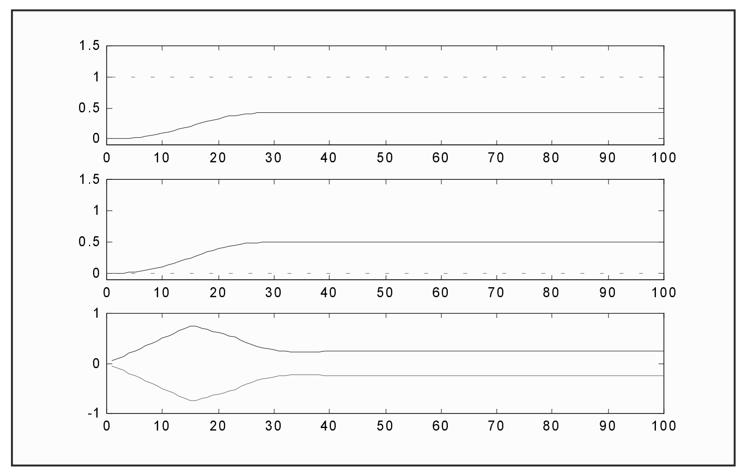


Identification with Random Inputs



First Order Model

MPC for first Order



How to Select Better Inputs :

$$\hat{X} = X + E, \qquad \sigma\left(\hat{X}\right) = \sigma\left(X\right) + \left\|E\right\|$$

Need to make $\sigma_{\min}\left(X\right)$ as big as possible.

Make outputs as uncorrelated as possible.

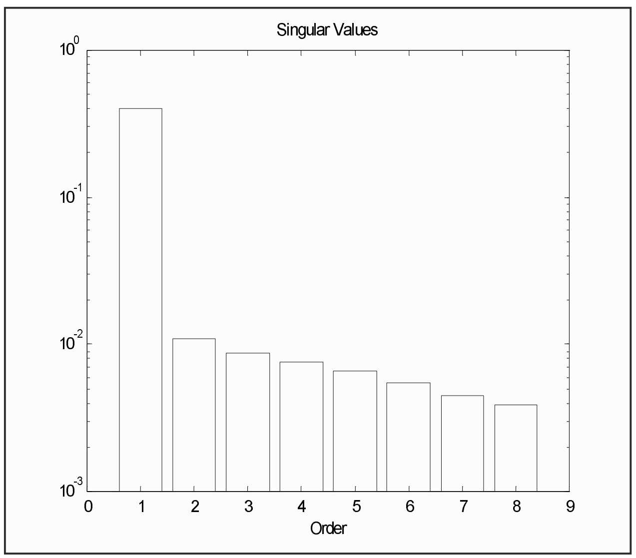
$$Y = G(s)\,U \longrightarrow Y = \mathrm{USV}\,U \longrightarrow Y = \mathrm{US}\,\Xi$$

$$\frac{\left|\xi_n\right|}{\left|\xi_1\right|} = \kappa$$

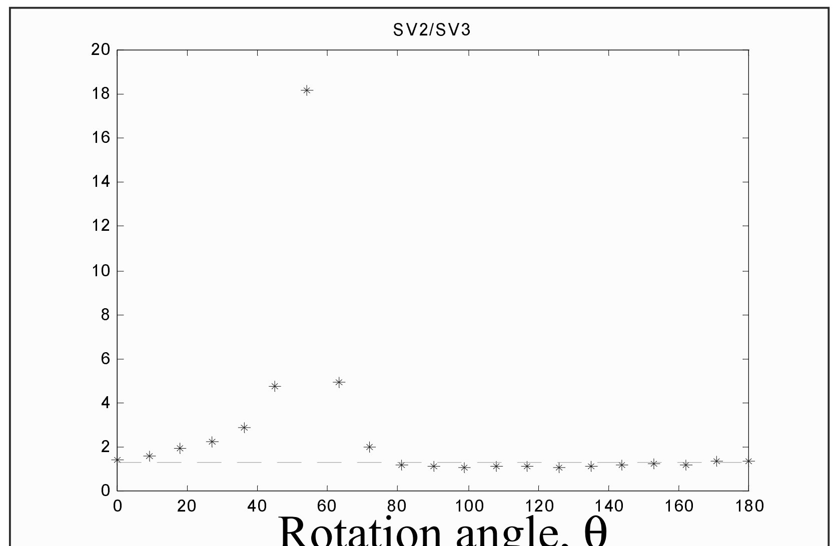
κ not known, Ξ not known

SI for Distillation Example: Different input designs

- **Singular values for different rotations of inputs (Rotation matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$)**



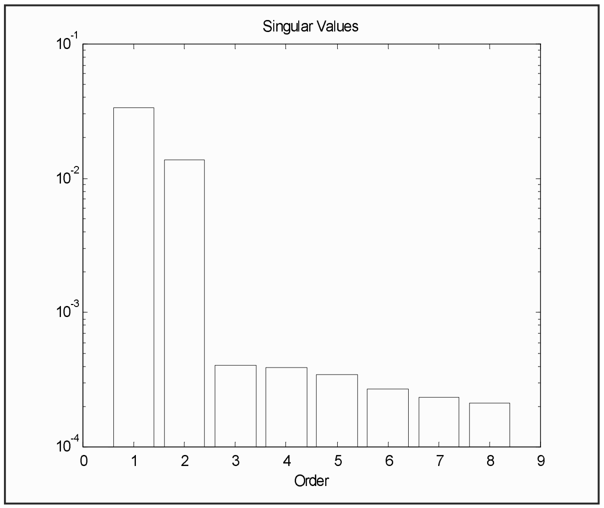
Singular Values for white noise



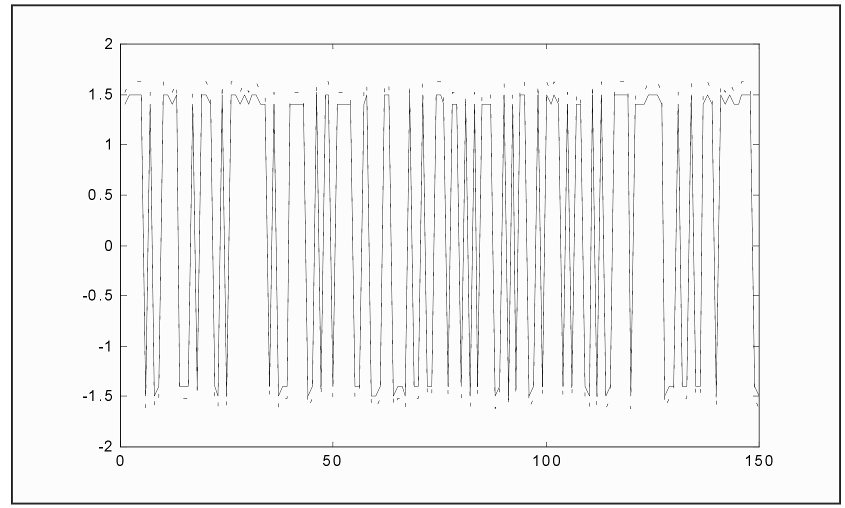
σ(2)/σ(3)

DISTILLATION EXAMPLE….

Singular Value Plot



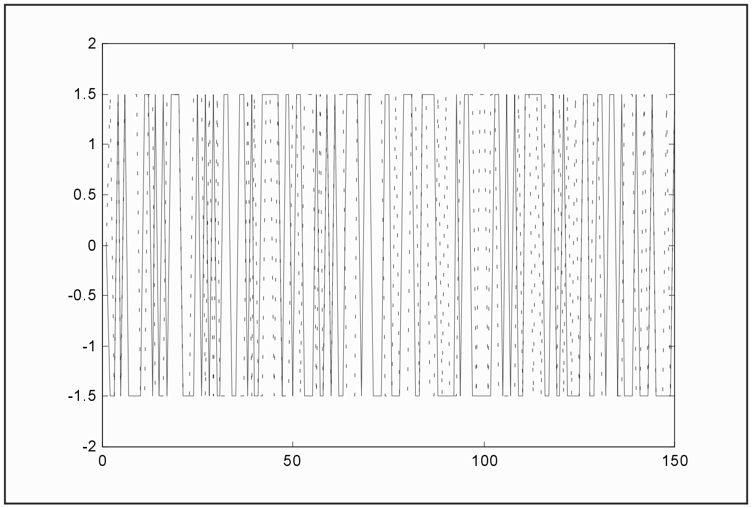
INPUTS



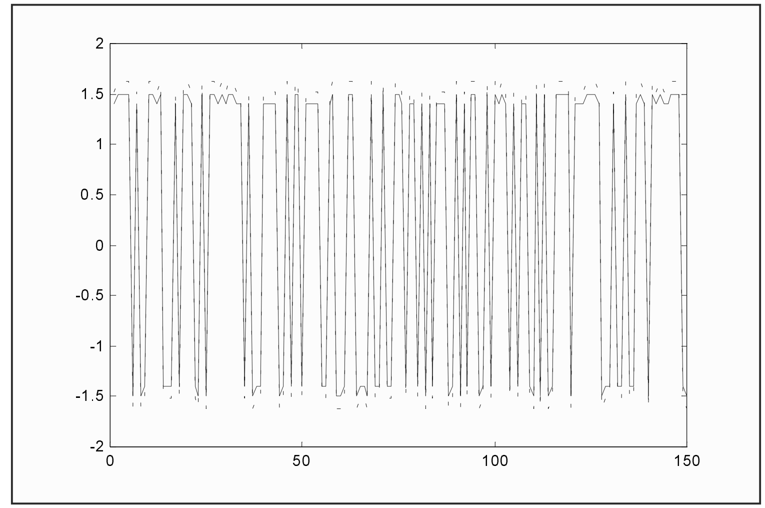
- State Space Dimension = 2
- Identified System Poles - 0.9744 , 0.7133
- "Real" System Poles - 0.9745 , 0.7166

DISTILLATION EXAMPLE….

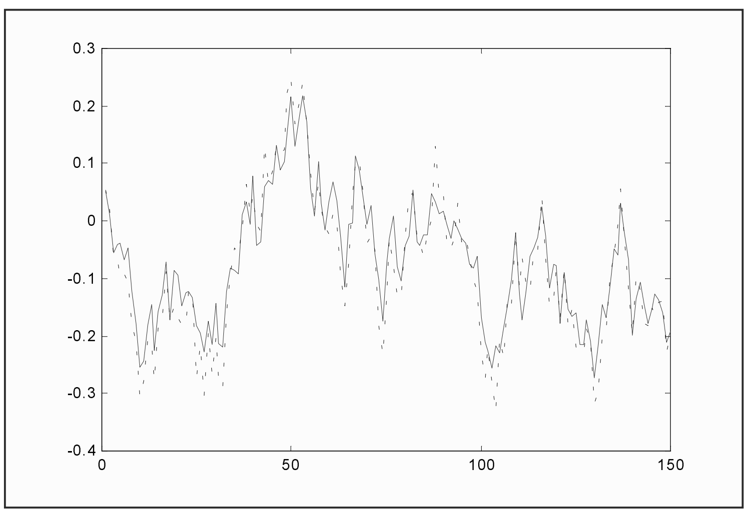
Random inputs



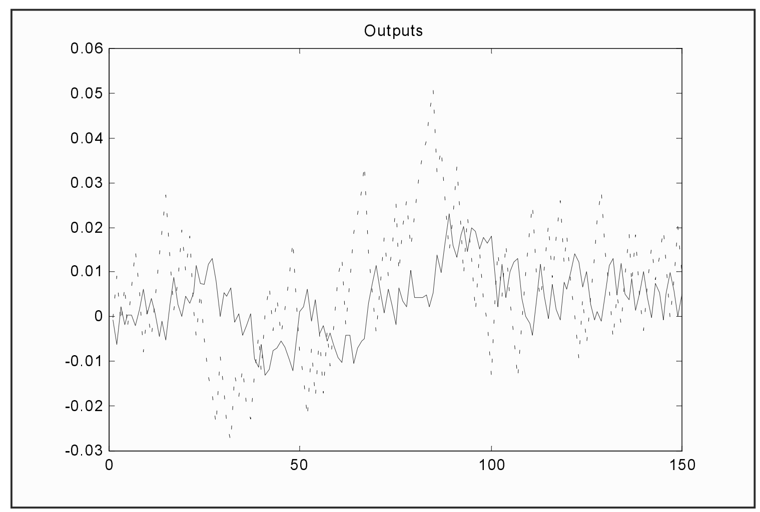
Rotated random inputs



Outputs



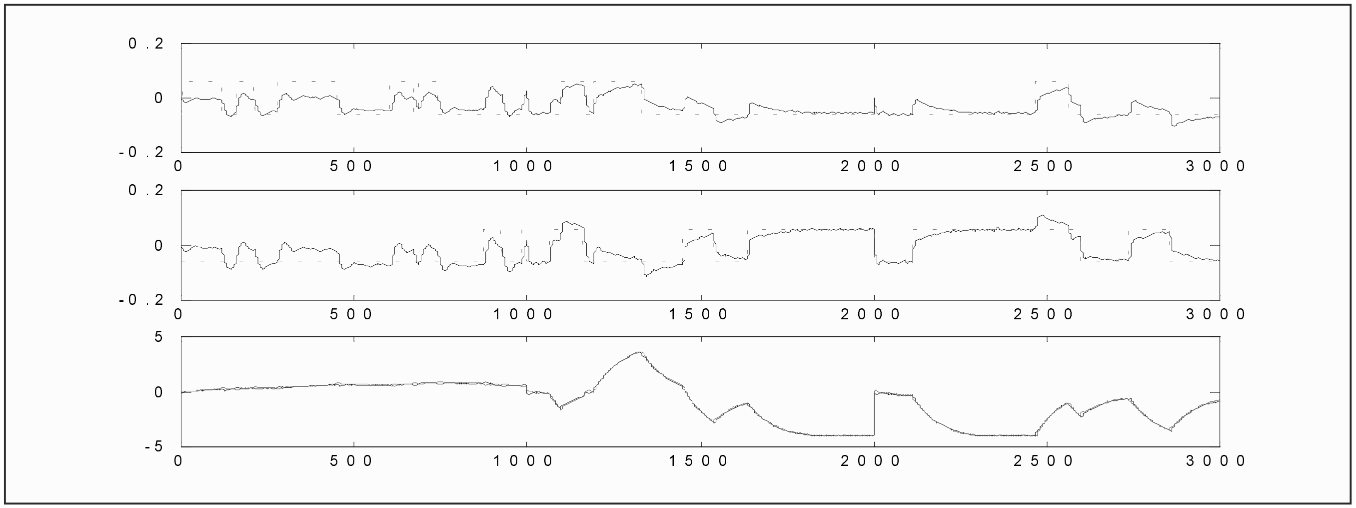
Outputs



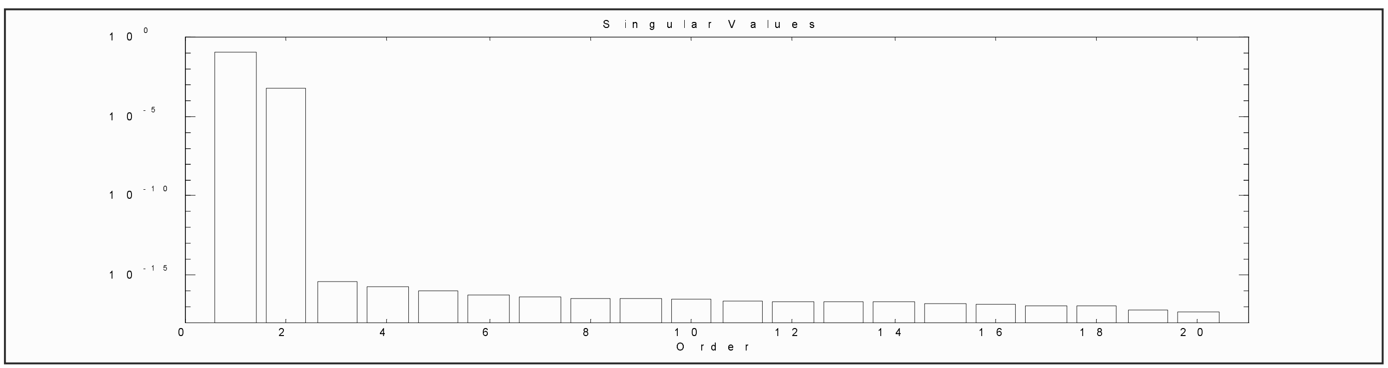
CLOSED LOOP INPUT DESIGN

- Closed Loop Identification - cheaper, safer.
- Outputs as uncorrelated as possible.
- Easily accomplished in closed loop.
  - Inputs not directly accessible
  - Output Setpoints Orthogonal

DISTILLATION EXAMPLE



Input-Output data



Model order = 2

Conclusions

- MPC can be extended to include closed-loop identification
- Limitations are not severe
- Potential uses in diagnosis

Acknowledgments

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