

Robust Offset Free Model Predictive Control

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Research Objective

- Develop a new robust MPC theory that stabilizes a system with a **polytopic model uncertainty** description Ω . $(A, B) \in \Omega$ if

$$A = \sum_{i=1}^I \mu_i A_i, \quad B = \sum_{i=1}^I \mu_i B_i, \quad \sum_{i=1}^I \mu_i = 1, \quad \mu_i \geq 0$$

- Add a **tree trajectory** to the regulator to simulate time-varying uncertainties.
- Modify the controller formulation to include integral control **offset free** non-zero set point tracking.
- Evaluate the closed-loop stability.

Infinite Horizon Regulator

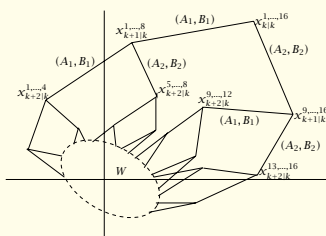


Figure 1: Regulator tree trajectory for $I = 2$ and $N = 4$.

$$\min_{\pi_k^l} \max_{l=1, \dots, N_I} \Phi_k^l$$

subject to

$$x_{k+j+1|k}^l = A_{v_l(j+1)} x_{k+j|k}^l + B_{v_l(j+1)} u_{k+j|k}^l + z_{k+j|k}^l$$

$$x_{k+j|k}^l \in W \quad \forall j \geq N, \quad x_{k+j|k}^l \in X, \quad u_{k+j|k}^l \in U$$

Remarks:

- Φ_k^l is the performance objective for trajectory l at time k .

$$\Phi_k^l = x_{k+N|k}^{lT} F x_{k+N|k}^l + \sum_{j=0}^{N-1} x_{k+j|k}^{lT} Q x_{k+j|k}^l + u_{k+j|k}^{lT} R u_{k+j|k}^l$$

- W is the invariant terminal region in which the control policy $u_{k+j|k}^l = K x_{k+j|k}^l$ is robustly stabilizing.
- $F = F^T > 0$ is the final state penalty matrix.

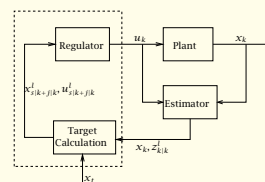
$$F = \sum_{j=N}^{\infty} x_{k+j|k}^{lT} Q x_{k+j|k}^l + u_{k+j|k}^{lT} R u_{k+j|k}^l$$

in which $Q \geq 0$ and $R > 0$.

- $v_l \in \mathbb{R}^N$ is the row vector containing the model indices necessary to simulate the time-varying uncertainties.
- N_I is the maximum number of possible trajectories, $N_I = I^N$.
- $(A, B) \in \Omega$ is robustly stabilizable if there exists K and F such that

$$F - Q - K^T R K - (A_i + B_i K)^T F (A_i + B_i K) \geq 0 \quad \forall i = 1, \dots, I$$

Robust MPC With Target Calculation



$$\min_{u_{k+j|k}^l} \max_{l=1, \dots, N_I} (x_{k+j|k}^l - x_t)^T Q_s (x_{k+j|k}^l - x_t)$$

subject to

$$x_{k+j|k}^l = A_{v_l(j)} x_{k+j|k}^l + B_{v_l(j)} u_{k+j|k}^l + z_{k+j|k}^l$$

$$u_{k+j|k}^l \in U \quad x_{k+j|k}^l \in X$$

Remarks:

- Assume state is measurable.
- The integral model is $z_{k+j|k}^l \in \mathbb{R}^n$, the disturbance value at time j in the prediction horizon with state measurement up to time k .
- To simulate time-varying uncertainties, $z_{k+j|k}^l$ is not constant but time-varying for $j > 0$.
- The time-varying trajectories for $z_{k+j|k}^l$ in the regulator cause $x_{k+j|k}^l \in \mathbb{R}^n$ and $u_{k+j|k}^l \in \mathbb{R}^m$, the steady-state trajectories with measurement up to time k , to be time-varying as well.
- Robust stability is dependent on the regulator dynamics and the target calculation.
- Offset free integral control for non-zero set point tracking can be accomplished if there exist \tilde{K} and \tilde{F} such that

$$F - \tilde{Q} - \tilde{K}^T \tilde{R} \tilde{K} - (\tilde{A}_i + \tilde{B}_i \tilde{K})^T \tilde{F} (\tilde{A}_i + \tilde{B}_i \tilde{K}) \geq 0 \quad i = 1, \dots, I$$

in which

$$\begin{bmatrix} u_{k+j+1|k}^l \\ x_{k+j+1|k}^l \end{bmatrix} = \tilde{A}_{v_l(j+1)} \begin{bmatrix} u_{k+j|k}^l \\ x_{k+j|k}^l \end{bmatrix} + \tilde{B}_{v_l(j+1)} \begin{bmatrix} u_{k+j|k}^l \\ u_{k+j|k}^l \end{bmatrix}$$

$$\tilde{Q} = \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix} \quad \tilde{R} = \begin{bmatrix} R & -R \\ -R & R \end{bmatrix} \quad \tilde{K} = \begin{bmatrix} I & 0 \\ I & K \end{bmatrix} \quad \tilde{F} = \begin{bmatrix} I & 0 \\ 0 & F \end{bmatrix}$$

- An LMI-based optimizer is used to solve for \tilde{K} and \tilde{F} subject to the equality constraints.

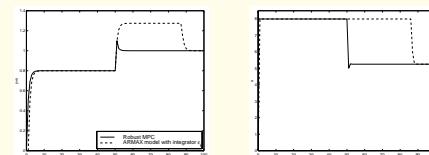
Constraint Saturation Example

$$G_1 = \frac{0.1}{s+10} \quad G_2 = \frac{1}{s+10}$$

$$y(s) = G_1 u(s)$$

$$-\infty \leq u_k \leq 8$$

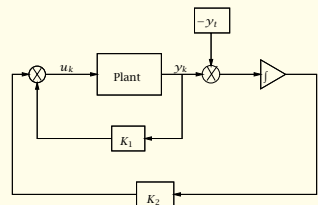
$$-\infty \leq y_k \leq \infty$$



Remarks:

- Model 2 is the plant and $y_t = 1$.
- The input constraint causes the set point to be unreachable.
- A disturbance at time $k = 50$ enters the system. The set point is reachable.
- Both robust MPC with target calculation and integral control with ARMAX models successfully reject the disturbance and reach the set point.
- But integral control with the ARMAX model exhibits **windup behavior** by delaying decreasing u_k from its constraint.

Integral Control With ARMAX Models



Remarks:

- The process models are Auto Regressive Moving Average Exogenous Inputs (ARMAX) polytopic models.
- The state is defined as

$$x_k = [y_k^T, \dots, y_{k-n+1}^T, u_{k-n+1}^T, \dots, u_{k-1}^T, \epsilon_k^T]^T$$

- Integral control is achieved by taking control action on ϵ_k .

$$\epsilon_k = \sum_{j=0}^k (y_t - y_j)$$

- K_1 and K_2 are the feedback gains on the extended state and ϵ_k , respectively.
- Offset free integral control is achieved when $\epsilon_k \rightarrow 0 \Rightarrow y_k \rightarrow y_t$.
- Target calculation is unnecessary.

Conclusion and Future Work

Conclusion

- Developed robust MPC theory that controls a time-varying uncertain system described by $(A, B) \in \Omega$.
- Modified the theory to include integral control that did not exhibit wind-up.
- Developed stability conditions that guaranteed **offset free** performance achieved **without windup**.

Future Work

- Replace state measurement with output measurement and state estimation.
- Study the model uncertainty description to see if it is possible to characterize the set $(A, B) \in \Omega$ for which robust offset free control is possible.