Robust Model Predictive Control

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Self Introduction

- Education

  1. Rice University
  2. University of Wisconsin - Madison

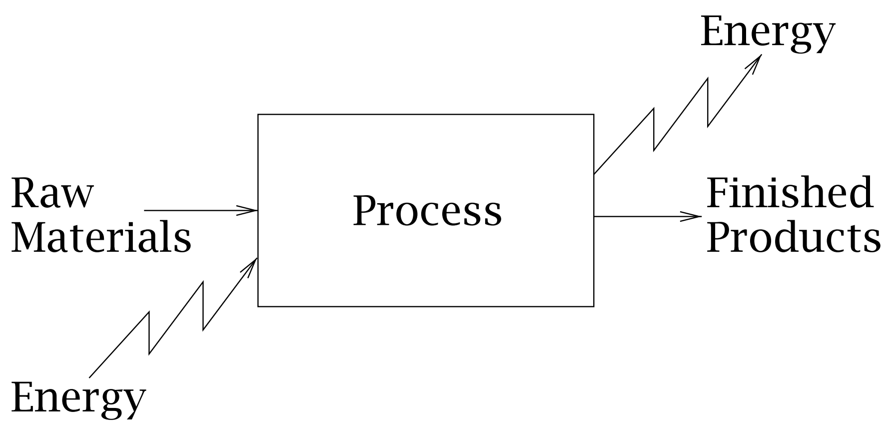
- Industrial Internships

  1. Shell Oil Company - Westhollow Technology Center
  2. Amoco Oil Company - Texas City Refinery
  3. Eastman Chemical Company

- Research Focus

  1. Process Control
  2. Model Predictive Control

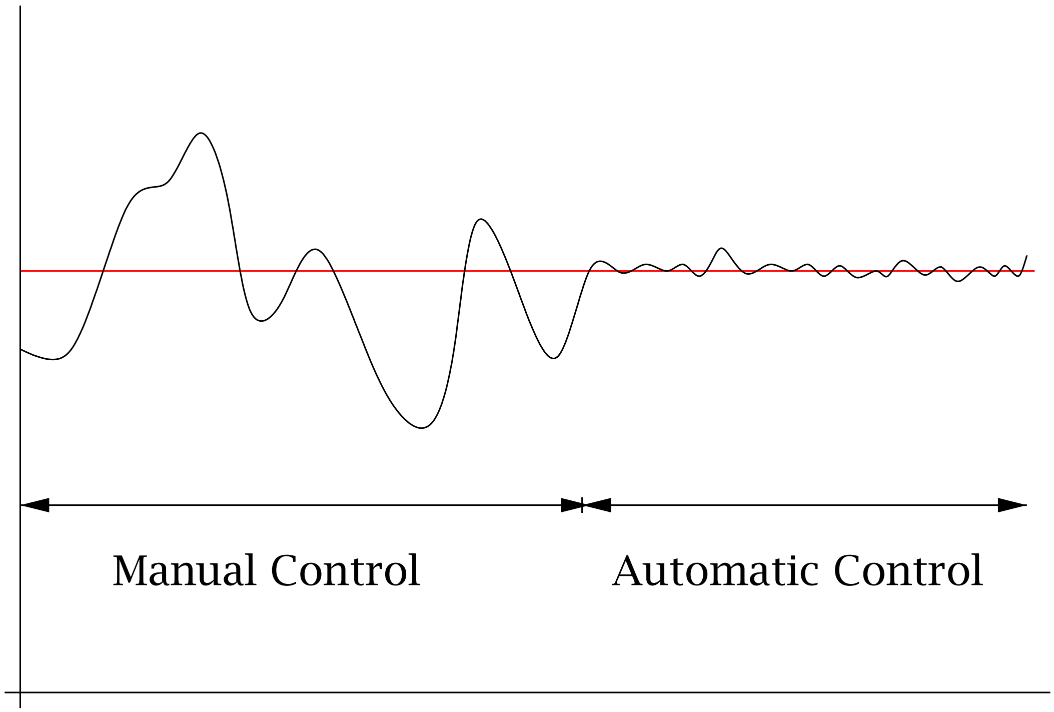
A Process



Basic Operating Principles

- Operate the processing units safely.
- Enhance the specified production rates.
- Enhance the product quality specifications.

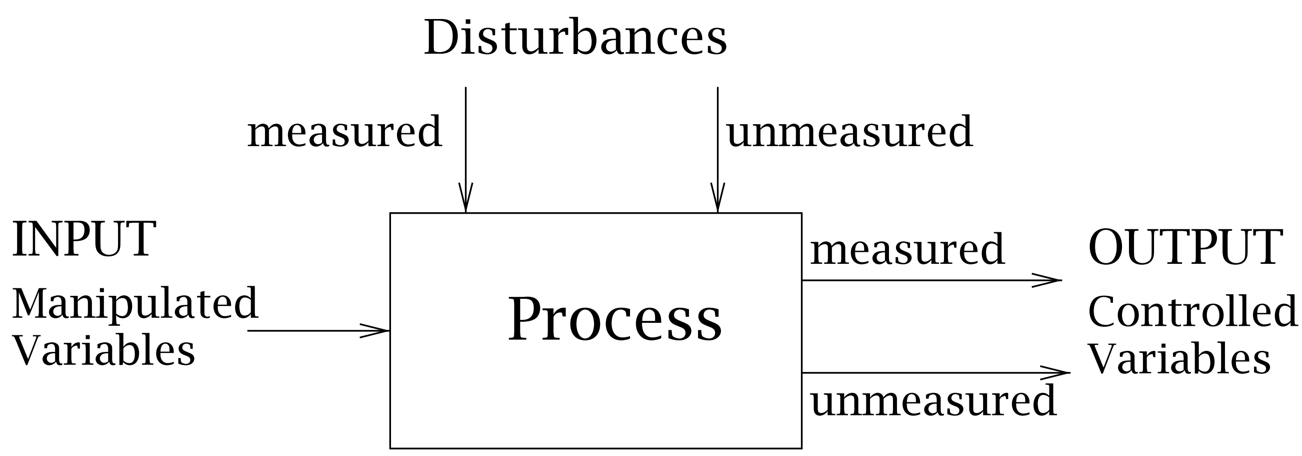
Motivation for Process Control



Automatic control

- decreases process variability.
- increases production of on-spec products.
- decreases operator intervention.

Responsibility of the Control System



- Measure the process output variables.
- Estimate the variables that are not measurable on-line.
- Decide on the corrective actions necessary
  1. to maintain the process at the current operating condition.
  2. to change the process from the current operating condition to another.
- Efficiently implement the corrective actions on the process.

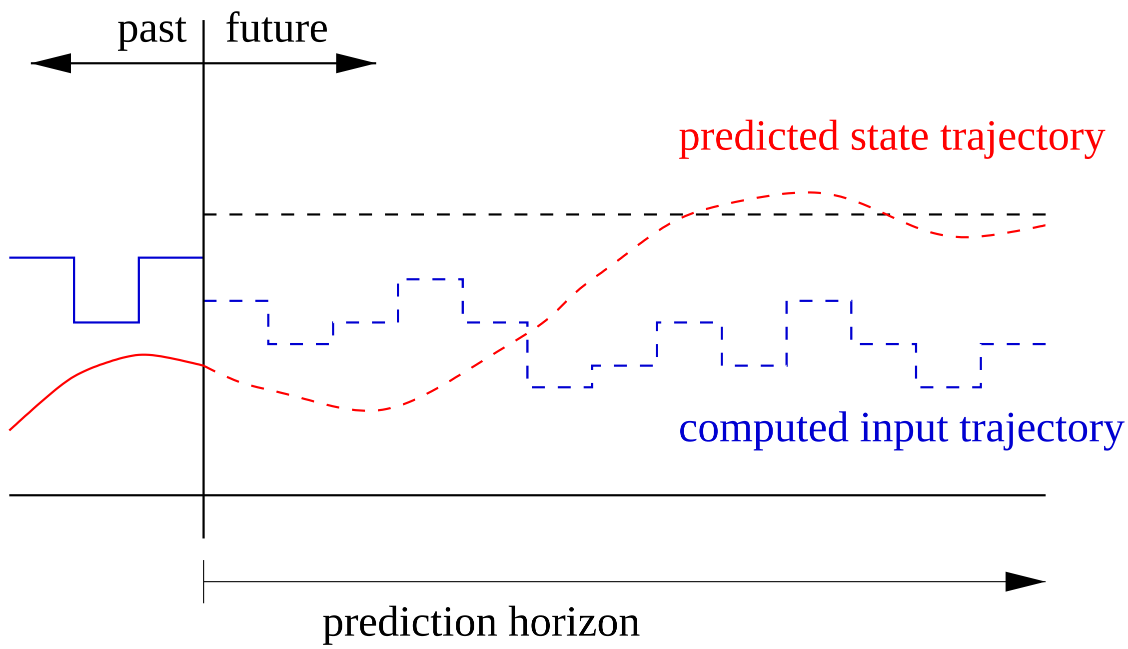
Outline

- Overview of Model Predictive Control (MPC)
- Motivation for robust MPC
- Robust MPC
  1. Benefits
  2. Cost
- Summary

Model Predictive Control

1. Regulator
    - Find the quickest route to reach the set point, the optimal operating condition.
2. Target calculation
    - Is the set point reachable for the given constraints?
    - If no, what are the best feasible steady-state values?
3. Estimator
    - Outputs that are not measurable.
    - Unmodeled disturbances.

Infinite Horizon Regulator



- A single process model is used to forecast the process behavior.

$$x_{k+1} = Ax_k + Bu_k$$
$$y_{k+1} = Cx_{k+1}$$

- The optimal input trajectory minimizes the difference between the forecasted and desired process behavior.

- Able to handle physical constraints explicitly.

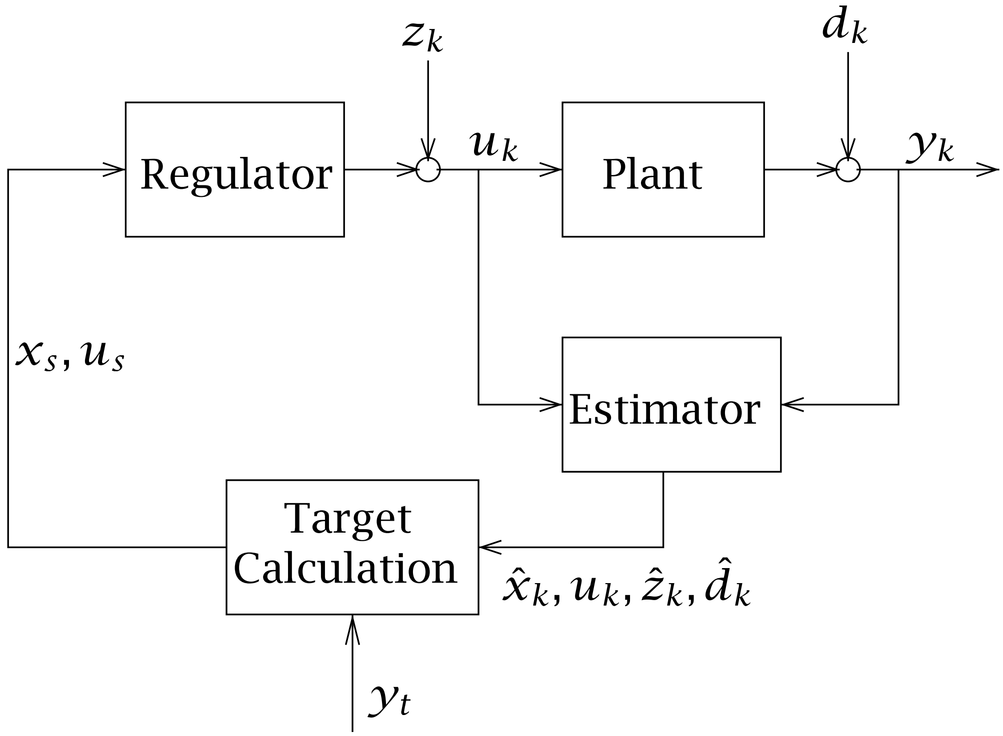
Industrial Impact of MPC

Industry	Applications
Refining	1500
Petrochemical	290
Chemicals	193
Pulp and Paper	45
Air Separation	5
Food Processing	41
Furnaces	42
Aerospace/Defense	13
Other	102
Total	2233

- There were 2233 industry applications of MPC in 1996.

- MPC is superior to traditional PID controllers because

  1. able to control both measurable and unmeasurable outputs.
  2. able to control multivariable systems more efficiently.
  3. able to handle constraints.



- The regulator, the estimator and the target calculation are all dependent on the process model (A, B, C).

- The controller performance is dependent on the accuracy of the process model.

- Plant model mismatch may cause

  1. sub-optimal controller performance.
  2. closed-loop instability.

- The MPC algorithm model:

$$A_m = 0.4, \quad B_m = 0.1, \quad C_m = 1$$

- The Plant: $A_p = 0.4$, $B_p = 0.7$, $C_p = 1$.

Nominal MPC Performance

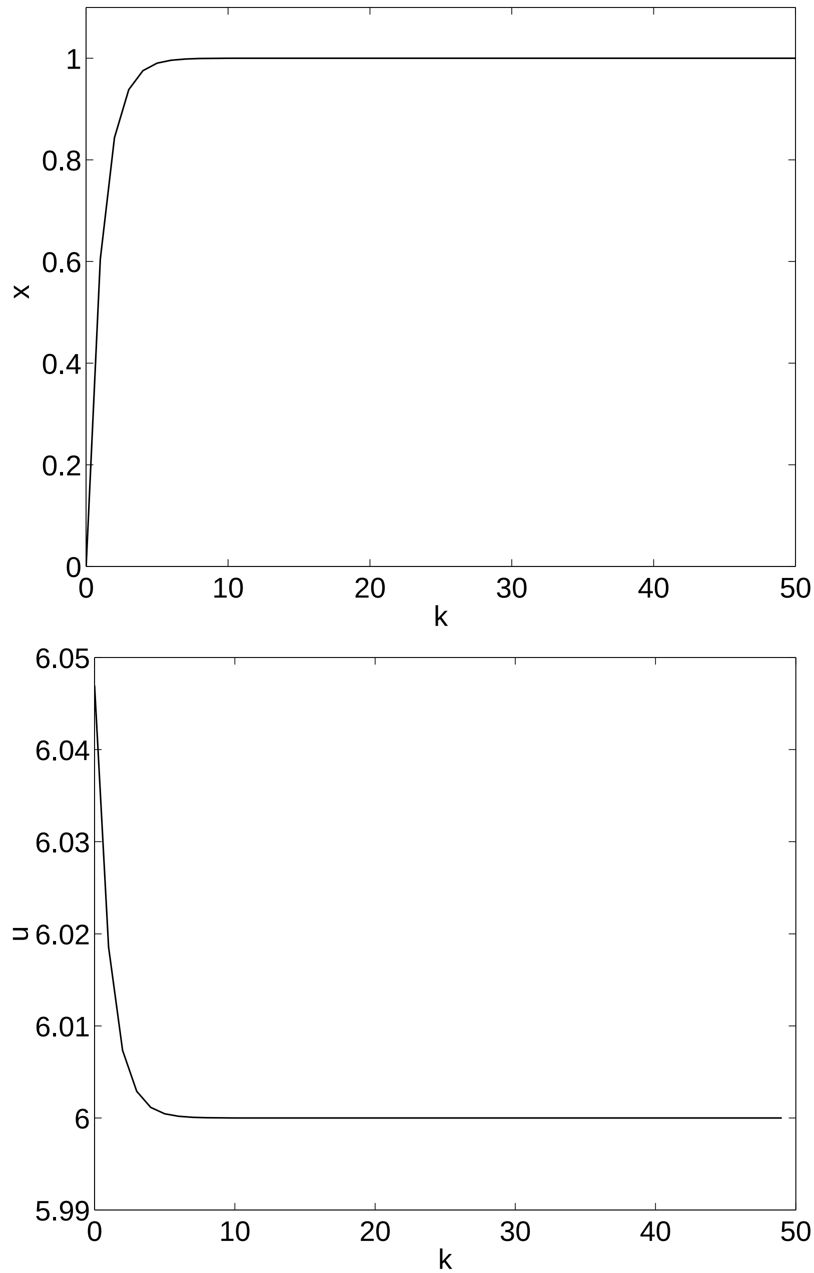


Figure 1: $A_p = A_m, B_p = B_m, C_p = C_m$

MPC Performance with Plant Model Mismatch

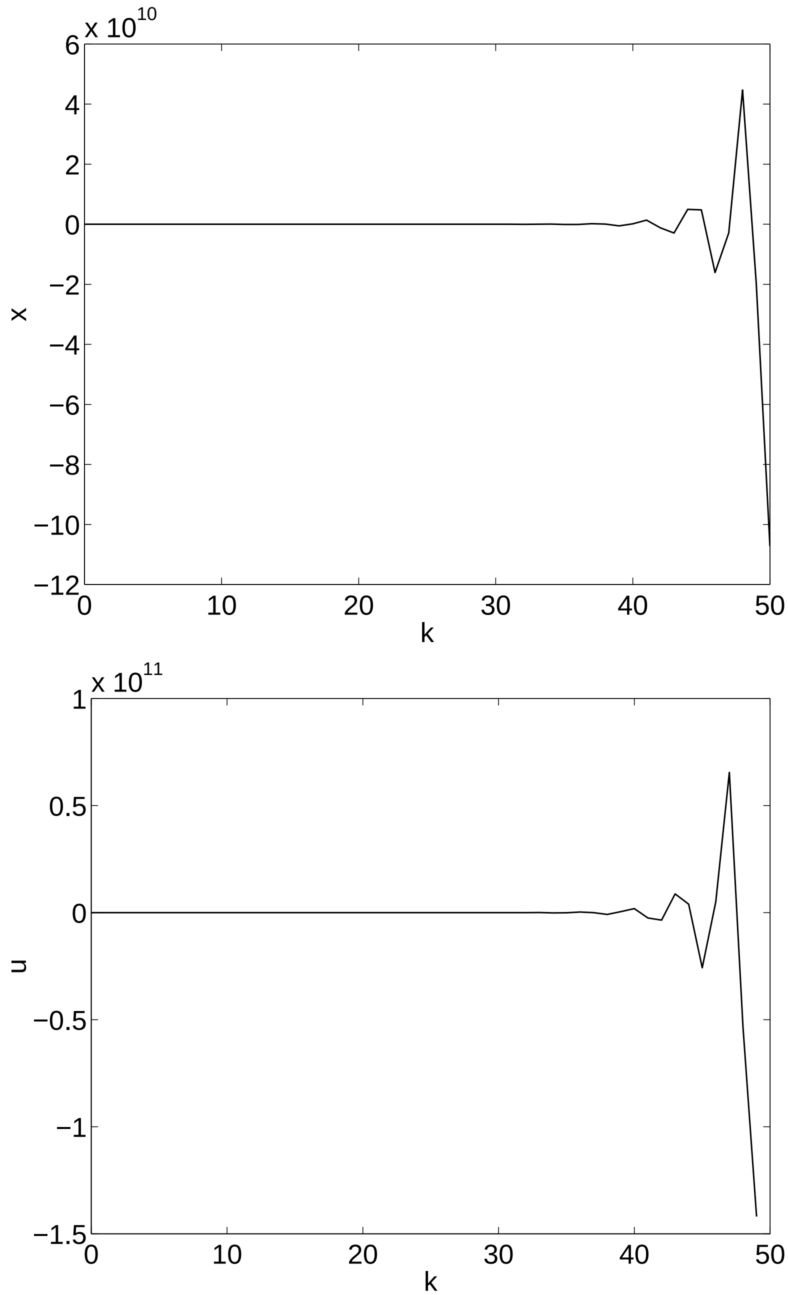
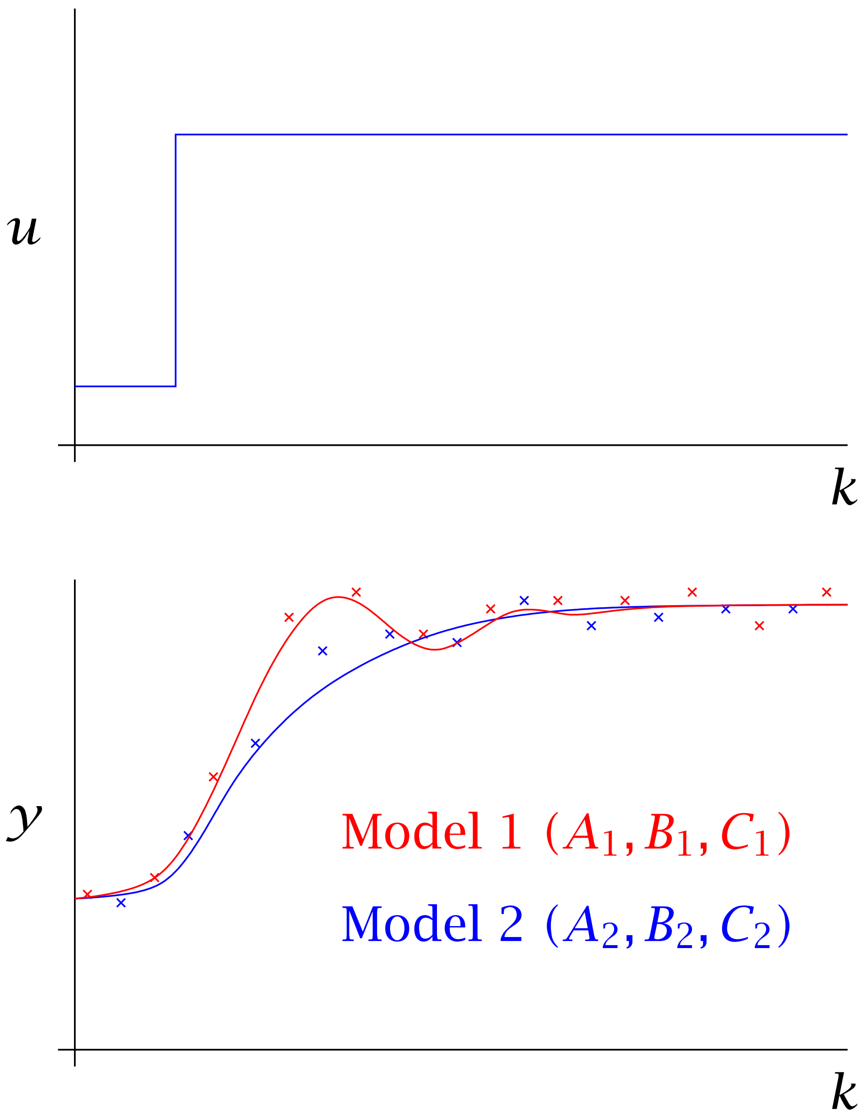


Figure 2: $A_p \neq A_m, B_p \neq B_m, C_p = C_m$

Motivation for Robust MPC

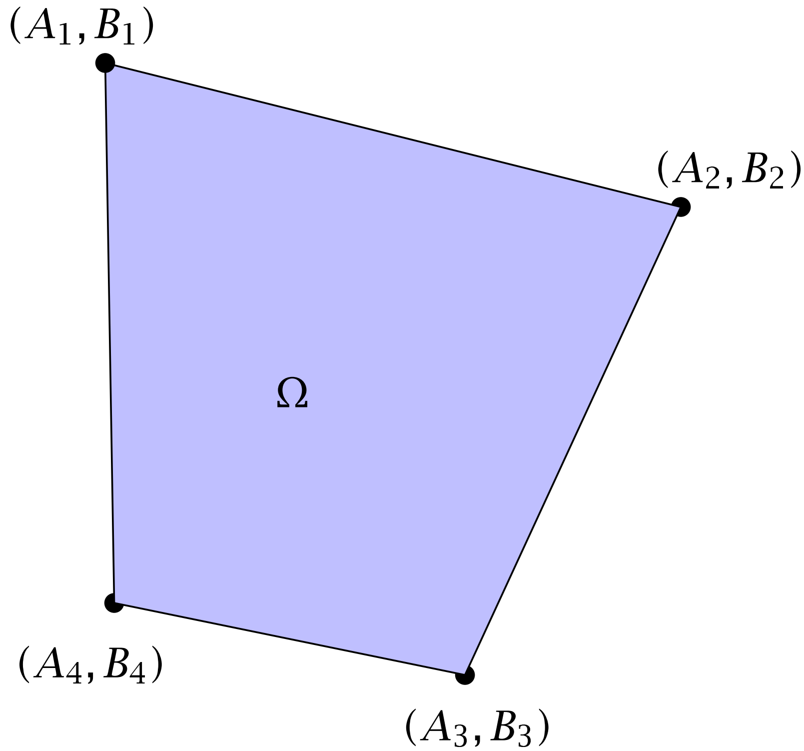
- Model uncertainty can cause closed-loop instability.
- Model uncertainty can be caused by:
  1. Unmodeled disturbances.
  2. Model identification errors.
  3. State estimation errors.
- Robust MPC explicitly accounts for model uncertainty in the controller design procedure.

Model Identification



- Model 1 used all data points.
- Model 2 used every other data point.

Model Uncertainty Description

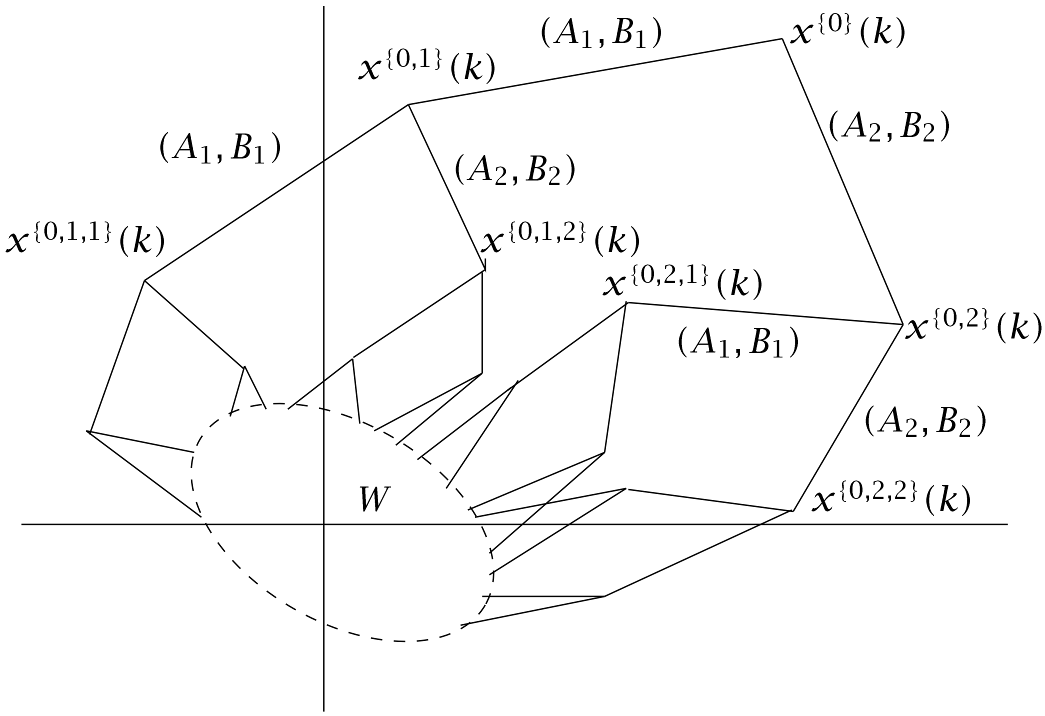


$(A, B) \in \Omega$ if

$$A = \sum_{i=1}^{I} \mu_i A_i, \qquad B = \sum_{i=1}^{I} \mu_i B_i$$

$$\sum_{i=1}^{I} \mu_i = 1, \qquad \mu_i \geq 0 \, \forall \; i = 1, \ldots, I$$

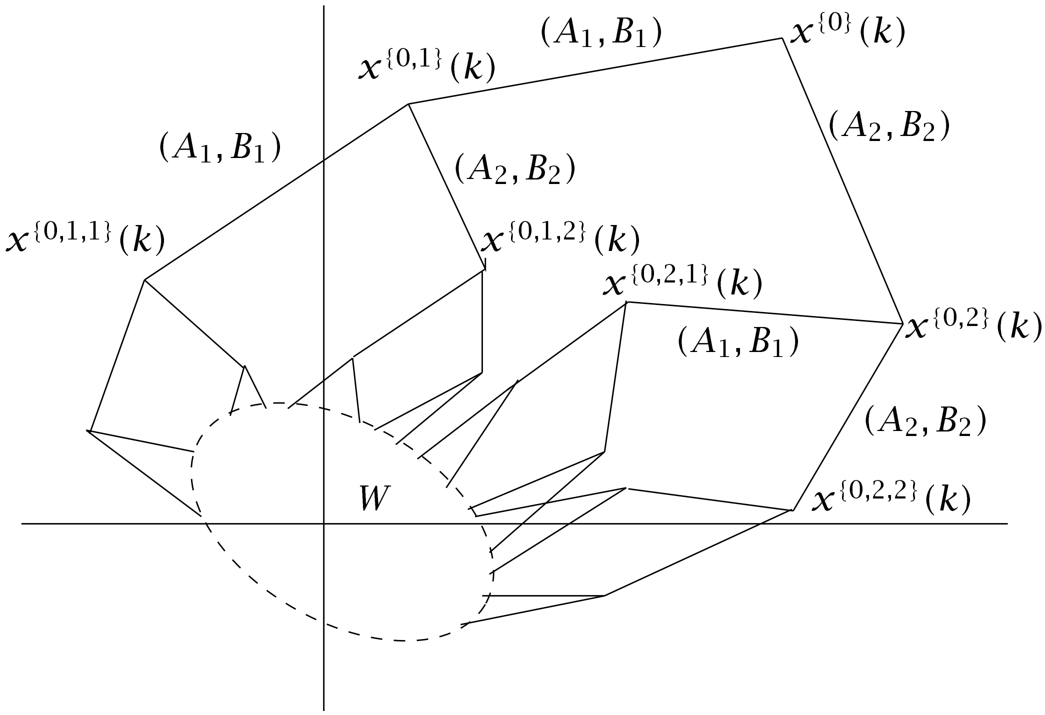
Problem Statement



- Forecasting the process behavior using many models.
- The number of branches is $L = I^N$.
- The number of decision variables is

$$\eta = \sum_{m=1}^{N-1} I^m$$

Example with $I = 2$ and $N = 4$



$$L = 2^4 = 16$$

$$\eta = 2^0 + 2^1 + 2^2 + 2^3 = 15$$

N has a larger effect on the number of decision variables than I.

Robust MPC

- Each branch has a cost, which is the sum of deviations of the forecasted behavior from the desired behavior.

- Optimal input trajectory minimizes the deviations for the branch with the "worst" or biggest cost.

- Controller is robustly stabilizing and guarantees state convergence to the origin for all $(A, B) \in \Omega$ if there exist K and F such that

$$F - Q - K^T R K - (A_i + B_i K)^T F (A_i + B_i K) \geq 0 \qquad \forall \ i = 1, \ldots, I$$

Advantages and Disadvantages

Features Present

1. Model uncertainty is explicitly accounted for.

2. Existence of robustly stabilizing K and F guarantees state convergence to the origin for all $(A, B) \in \Omega$.

Features Missing

1. K and F do not guarantee state convergence to non-zero set points.

2. Assume the state is measurable.

Proposed Solutions

1. Use ARMAX (Auto Regressive Moving Average Exogenous Input) polytopic model uncertainty description.

2. Add integrator.

ARMAX model with integrator

ARMAX Model

$$\begin{aligned} y(k+1) &= a_1 y(k) + \cdots + a_n y(k-n+1) \\ &+ b_1 u(k) + \cdots + b_n u(k-n+1) \end{aligned}$$

Model includes outputs that are measurable only.

$$x(k) = [y^T(k), \cdots, y^T(k-n+1), \cdots$$
$$u^T(k-n+1), \cdots, u^T(k-1), \epsilon^T(k)]^T$$

in which

$$\epsilon(k) = \sum_{j=0}^{k} (y_t - y(j))^2$$

Offset free control is achieved by taking control action on $\epsilon(k)$.

Constraint Saturation Example

$$\bar{y}(s) = G_i \bar{u}(s)$$
$$-\infty \leq u(k) \leq 8$$
$$-\infty \leq y(k) \leq \infty$$

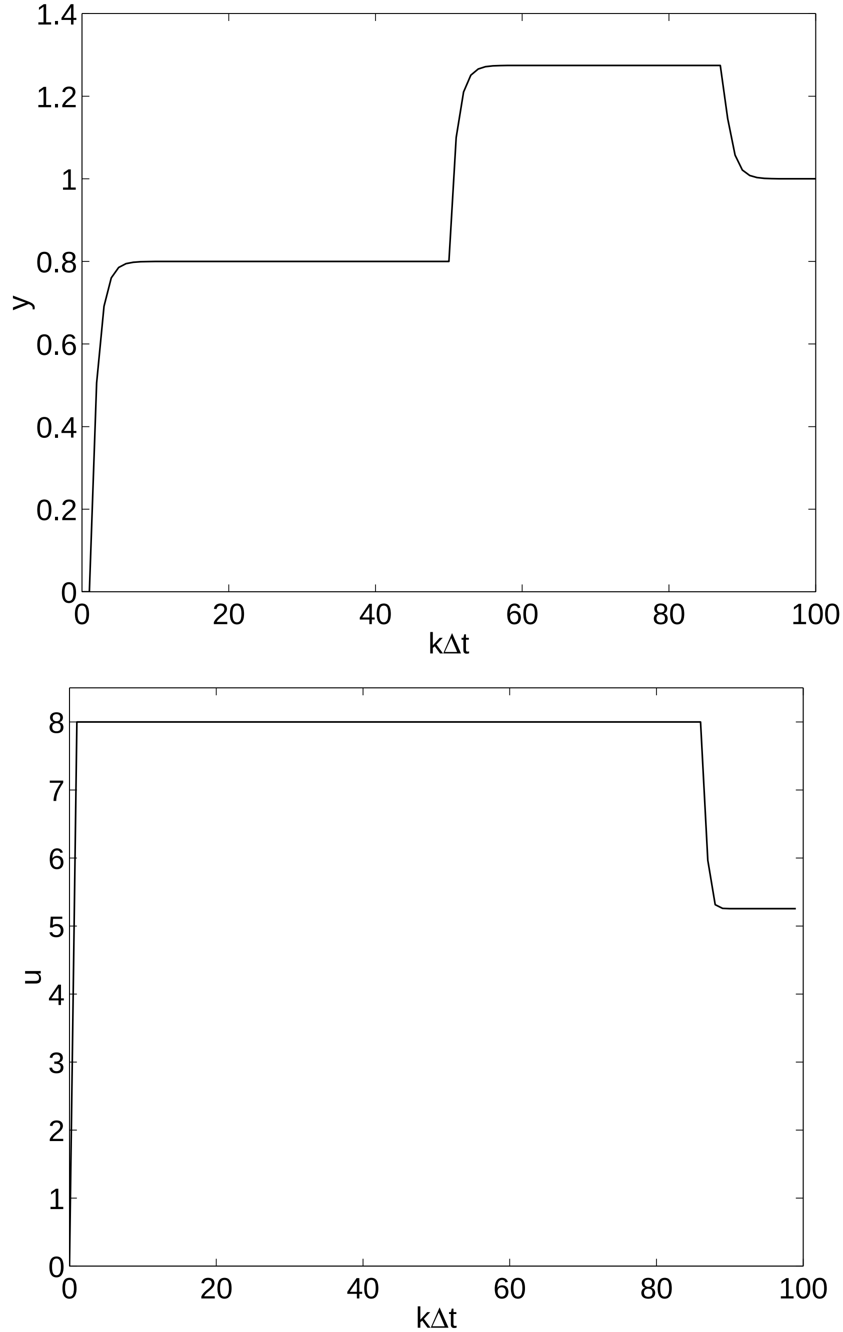
Model 1:

$$G_1 = \frac{0.1}{s + 10}$$
$$y(k+1) = 0.3679y(k) + 0.0063u(k)$$

Model 2:

$$G_2 = \frac{1}{s + 10}$$
$$y(k+1) = 0.3679y(k) + 0.0632u(k)$$

Constraint saturation example



Advantages and Disadvantages

Features Present

1. Controller achieves non-zero set point tracking.

2. State is measurable.

Features Missing

1. Uses input/output process models, which are less descriptive than the state-space models.

2. Controller exhibits wind-up in the presence of constraint saturation.

Proposed Solutions

1. Feedback and target calculation

2. State estimation

Feedback

- If the measurements are not equal to the model predictions, then the model predictions need to be updated.

- Given measurement at time k, the state predictions for time $k+1$ are $x^{\{0,1\}}(k)$, $x^{\{0,2\}}(k)$, ... , and $x^{\{0,I\}}(k)$.

- State measurement at time $k+1$ is $x(k+1)$.

- Update the state predictions with the disturbance models.

$$w_i^{\{0\}}(k+1) = w_i^{\{0\}}(k) + (x(k+1) - x^{\{0,i\}}(k))$$

- Process model with disturbance modeling

$$x^{\{\tau_j,i\}}(k) = A_i x^{\{\tau_j\}}(k) + B_i u^{\{\tau_j\}}(k) + w_i^{\{\tau_j\}}(k)$$

Target Calculation

Objective:

At each node, find the steady-state input that minimizes the state deviation from the set point for the worst model.

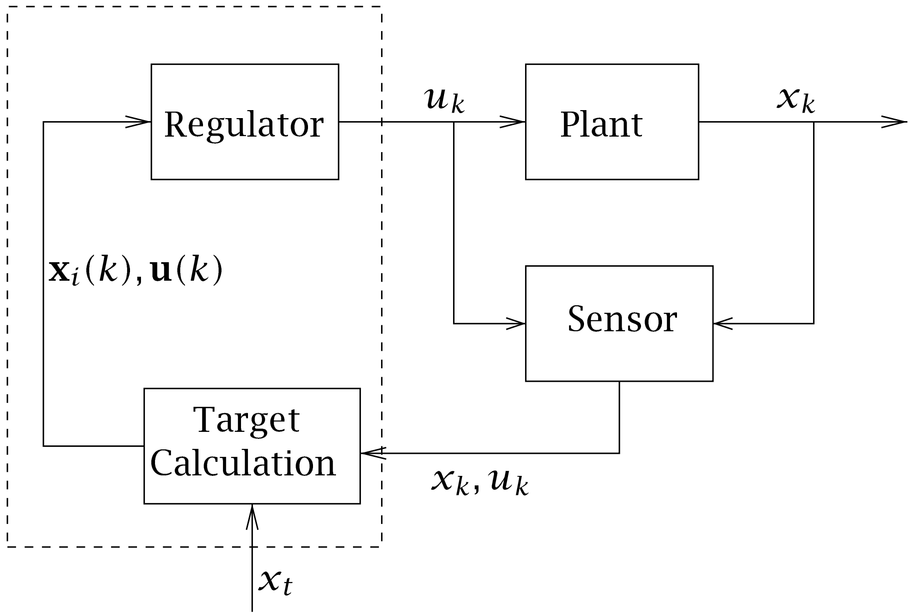
$$\min_{\mathbf{u}^{\{\tau_j\}}(k),\mathbf{x}_i^{\{\tau_j\}}(k)} \max_{i=1,\ldots,I} \Phi_i = (\mathbf{x}_i^{\{\tau_j\}}(k) - x_t)^T \mathbf{Q} (\mathbf{x}_i^{\{\tau_j\}}(k) - x_t)$$

subject to

$$\mathbf{x}_i^{\{\tau_j\}}(k) = A_i \mathbf{x}_i^{\{\tau_j\}}(k) + B_i \mathbf{u}^{\{\tau_j\}}(k) + w_i^{\{\tau_j\}}(k)$$

- The steady-state $\mathbf{u}^{\{\tau_j\}}(k)$ sends each model to a different $\mathbf{x}_i^{\{\tau_j\}}(k)$.
- System converges to x_t if all branches in the tree trajectory converge to x_t.

Regulator with Non-zero Set Point



- The tree trajectory models time-varying model uncertainty.
- The target calculation needs to be performed at every node in the tree trajectory.
- The controller stability is dependent on both the process dynamics and the target calculation.

Close-loop Stability Analysis

Offset free control can be achieved if there exist $\bar{K}$ and $\bar{F}$ such that

$$\bar{F} - \bar{Q} - \bar{K}^T \bar{R} \bar{K} - (\bar{A}_i + \bar{B}_i \bar{K})^T \bar{F} (\bar{A}_i + \bar{B}_i \bar{K}) \geq 0$$

$$\forall \; i = 1, \ldots, I$$

$$\bar{Q} = \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix} \qquad \bar{R} = \begin{bmatrix} R & -R \\ -R & R \end{bmatrix}$$

$$\bar{K} = \begin{bmatrix} I & 0 \\ I & K \end{bmatrix} \qquad \bar{F} = \begin{bmatrix} I & 0 \\ 0 & F \end{bmatrix}$$

New structure results from the inclusion of target calculation and process dynamics in the closed-loop stability analysis.

Controller performance comparison

Model 1:

$$x_{k+1} = 0.4x_k + 0.1u_k$$

Model 2:

$$x_{k+1} = 0.4x_k + 0.7u_k$$

Nominal MPC with plant model mismatch

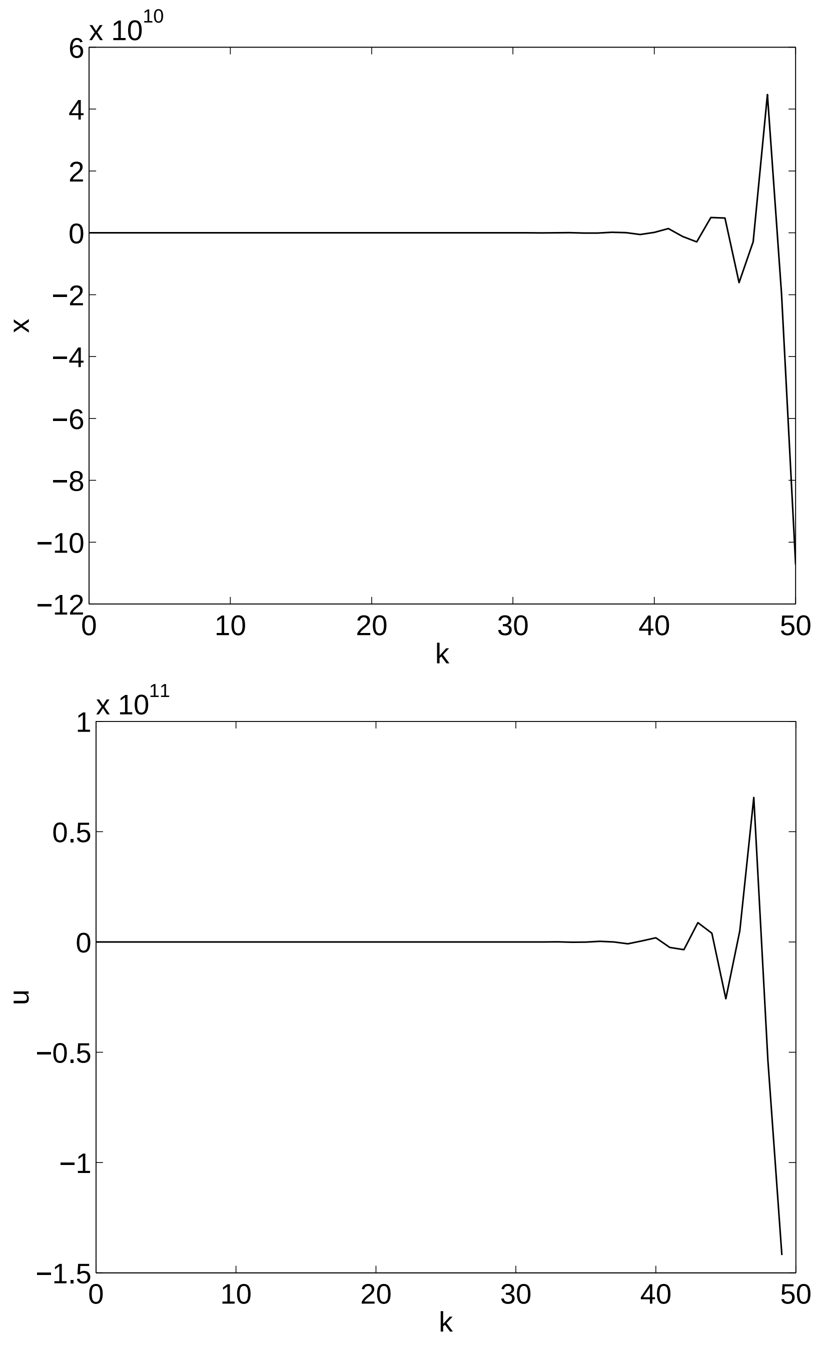


Figure 3: $A_m = 0.4$, $B_m = 0.1$ and $A_p = 0.4$, $B_p = 0.7$

Robust MPC

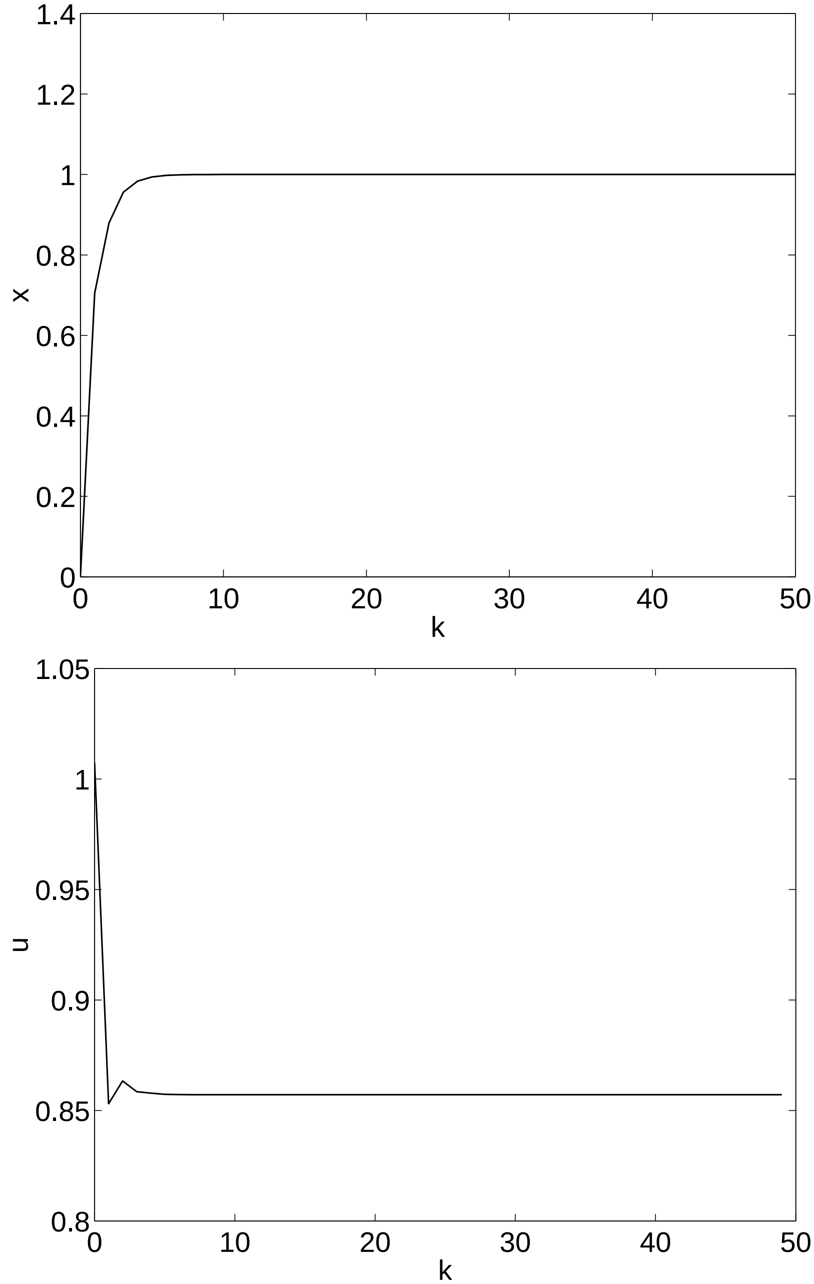


Figure 4: The plant is $A_p = 0.4$, $B_p = 0.7$.

Controller performance comparison

Model 1:

$$x_{k+1} = 0.4x_k + 0.1u_k$$

Model 2:

$$x_{k+1} = 0.4x_k + 0.4u_k$$

Robust and Nominal MPC performance comparison

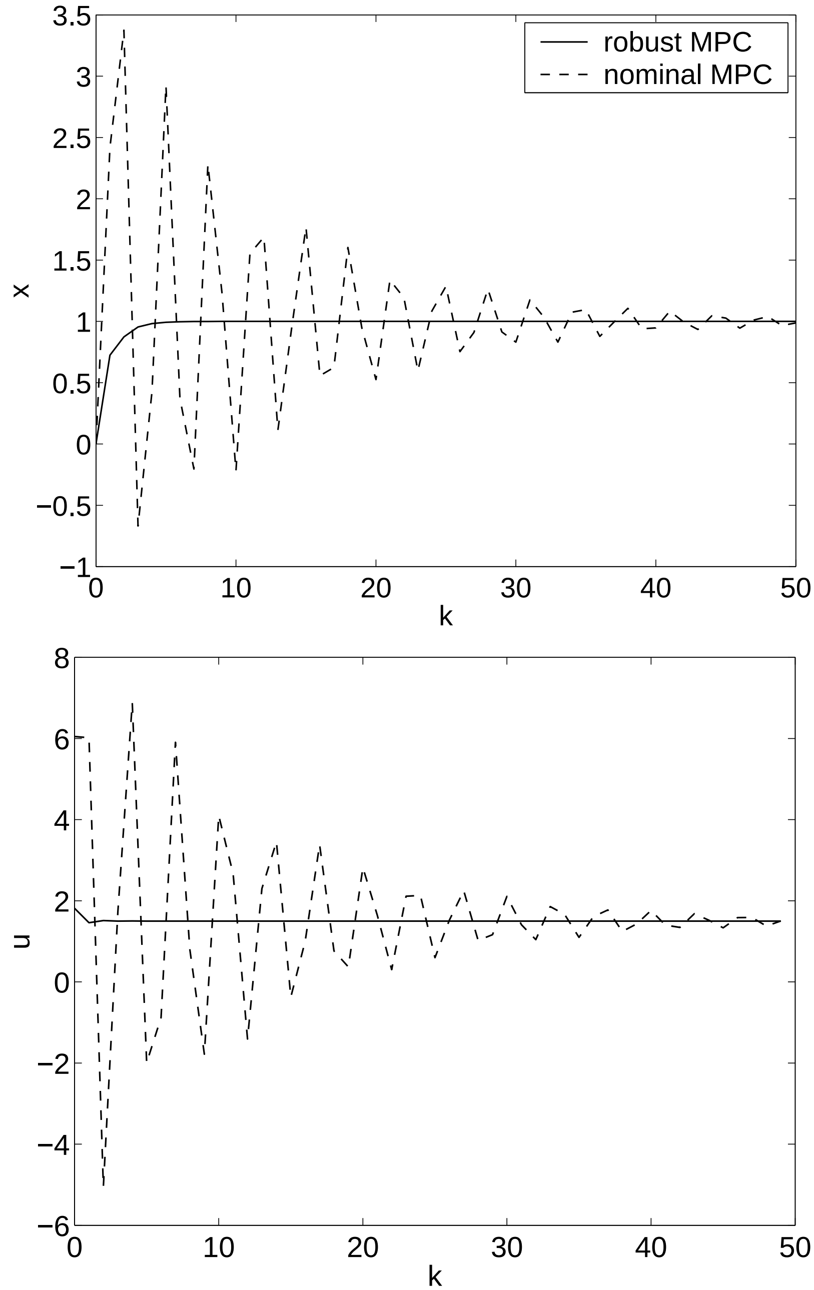


Figure 5: $A_m = 0.4$, $B_m = 0.1$ and $A_p = 0.4$, $B_p = 0.4$

Robust and Nominal MPC performance comparison

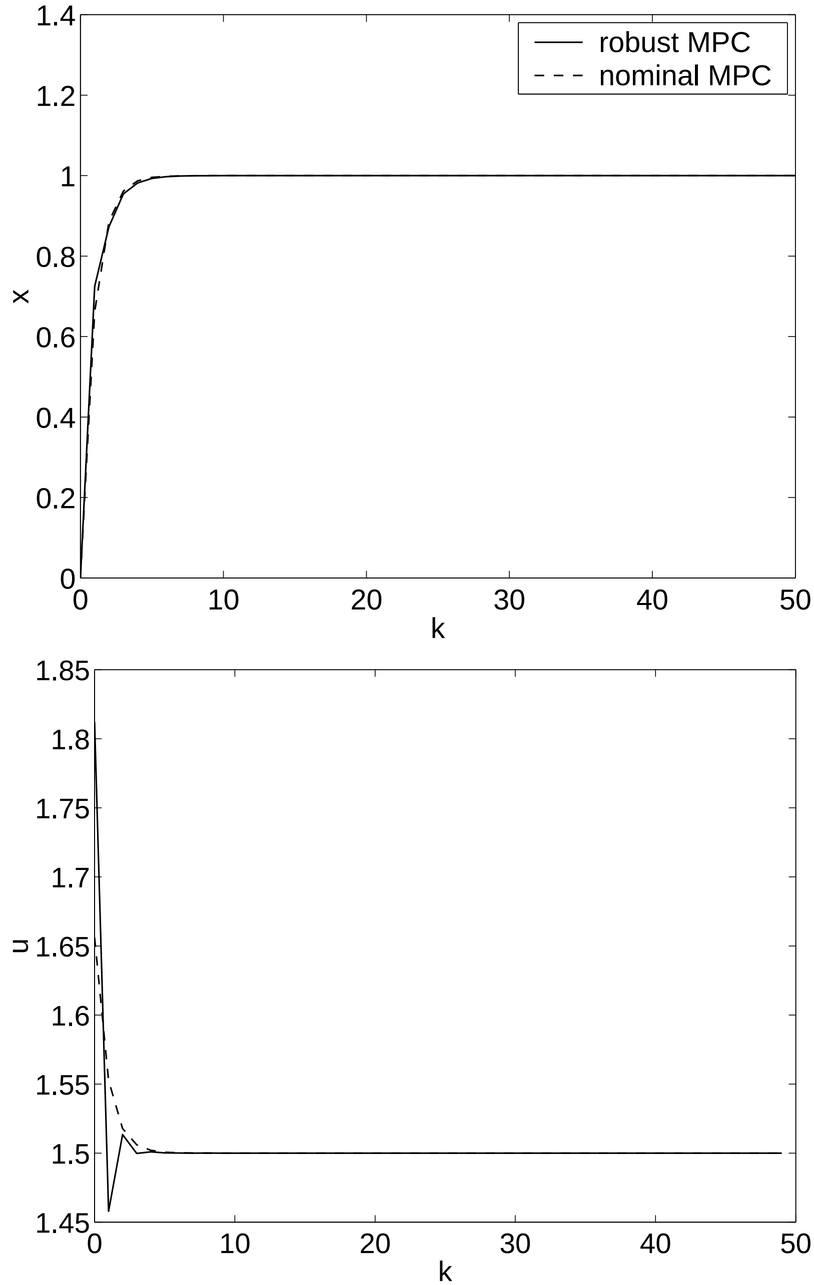


Figure 6: $A_m = A_p = 0.4$, $B_m = B_p = 0.4$

Robust and Nominal MPC performance comparison

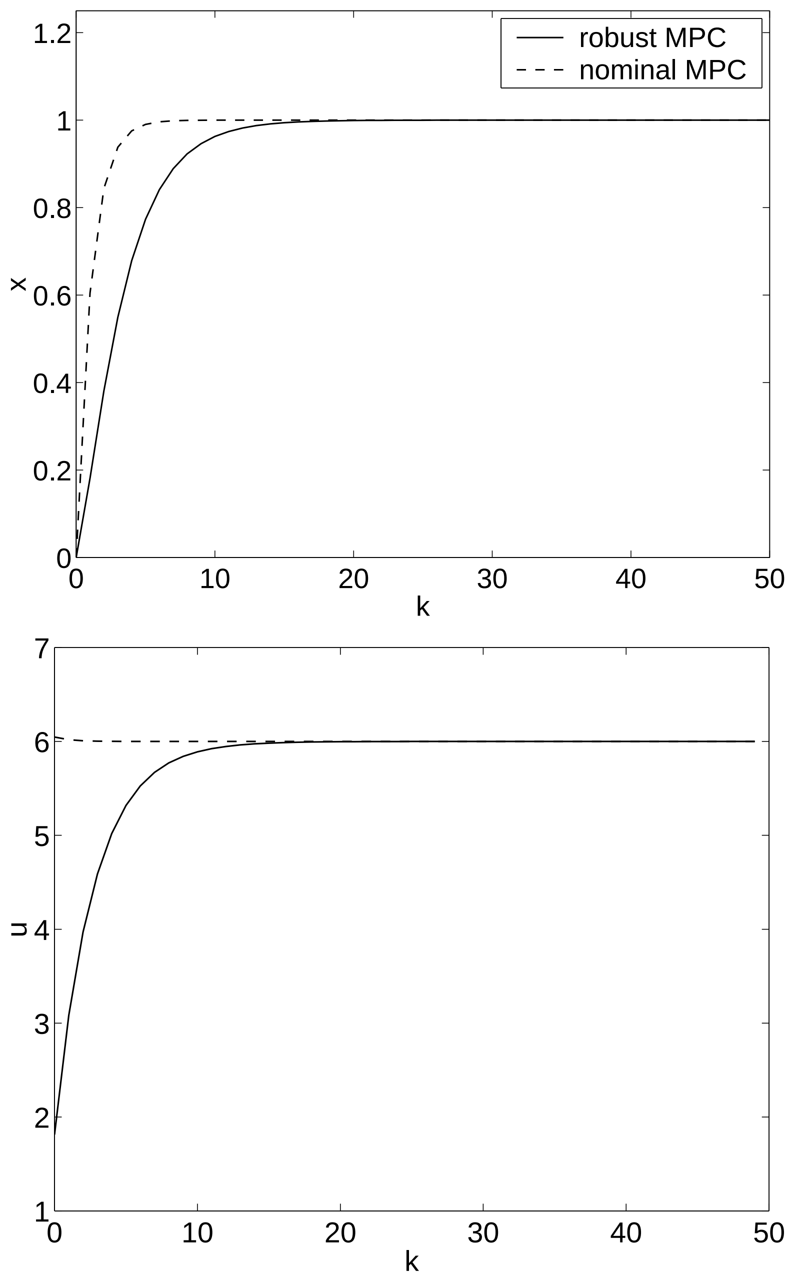
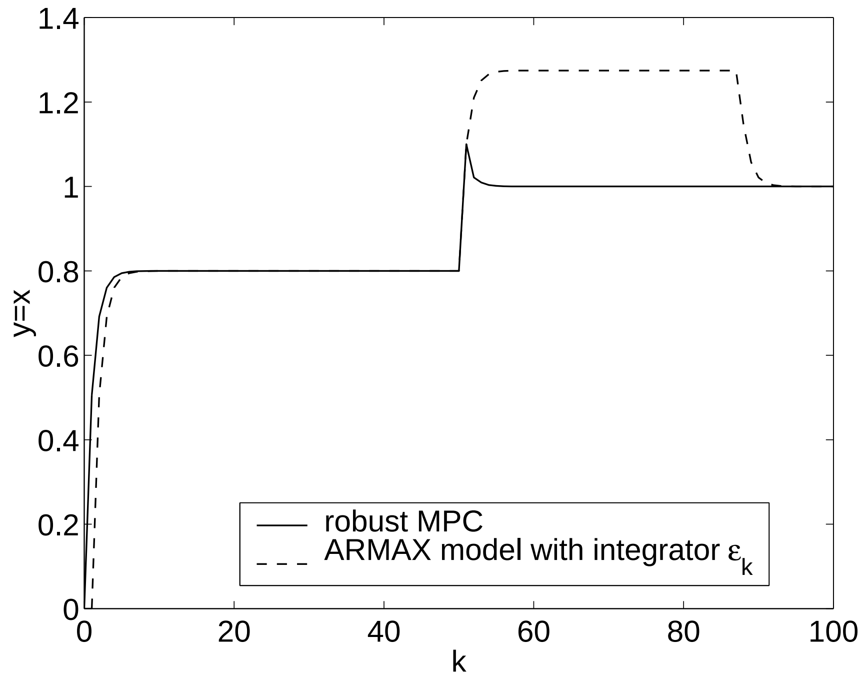
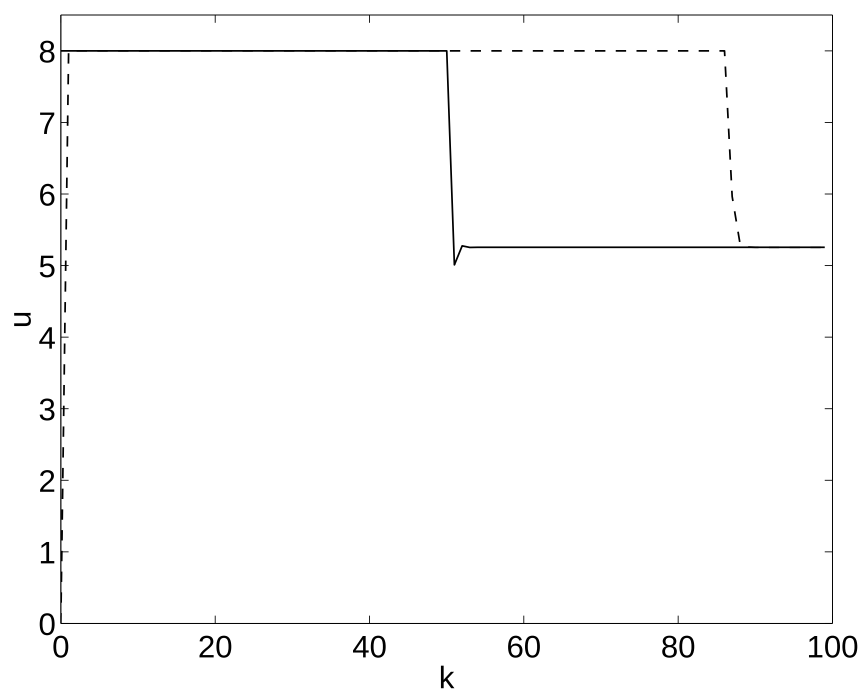


Figure 7: $A_m = A_p = 0.4$, $B_m = B_p = 0.1$

Constraint saturation example





Summary

Designed a robust MPC control theory that

1. explicitly accounts for model uncertainty.

2. uses many models to forecast process behavior.

3. able to handle constraints without wind-up in the presence of constraint saturation.

4. achieves offset free non-zero set point tracking control.

5. is computationally intensive but manageable for small processes.