Performance monitoring of the LQG System

Brian J. Odelson
Department of Chemical Engineering
University of Wisconsin-Madison

Madison, WI
September 26, 2001

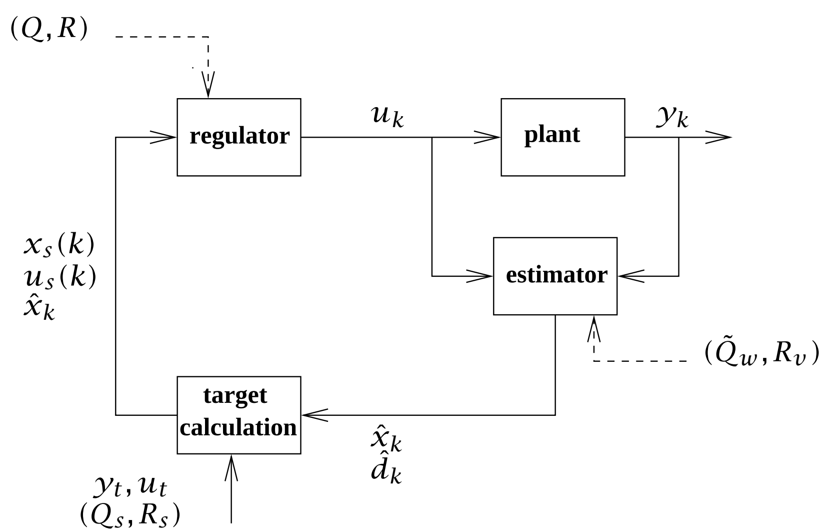
Outline

- Problem Statement
- Motivation
- New MPC Monitoring Layer
- Open Loop Techniques
- Closed Loop Techniques
- Robustness
- Conclusions
- Future Work
- Acknowledgements

Traditional MPC, no monitoring

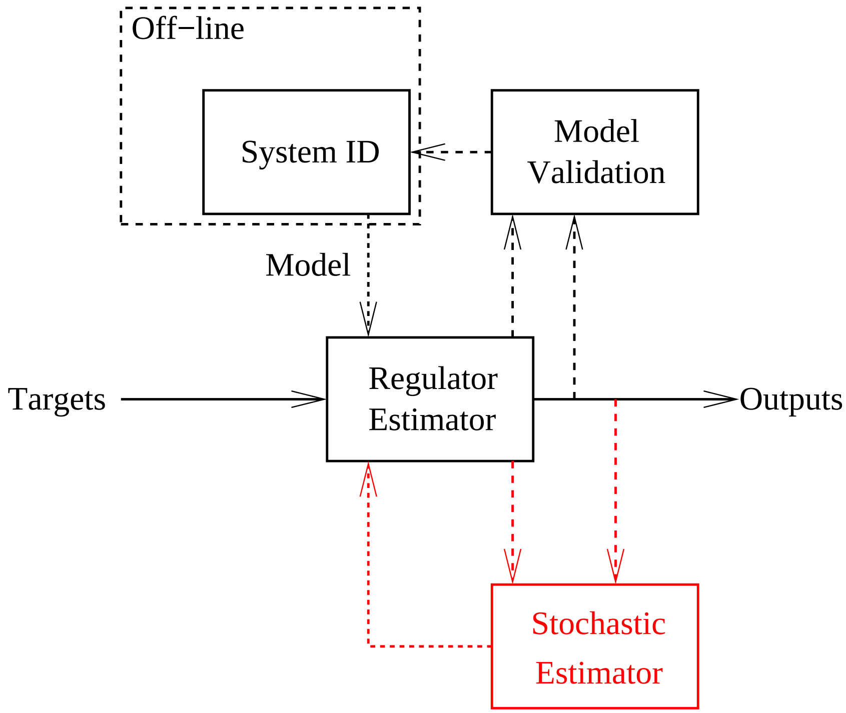
User defined parameters:

1. A,B,C,D - system model (subspace ID?)
2. Q,R,S - regulator penalties (performance objectives)
3. Q_w,R_v - estimator tuning (focus of talk)



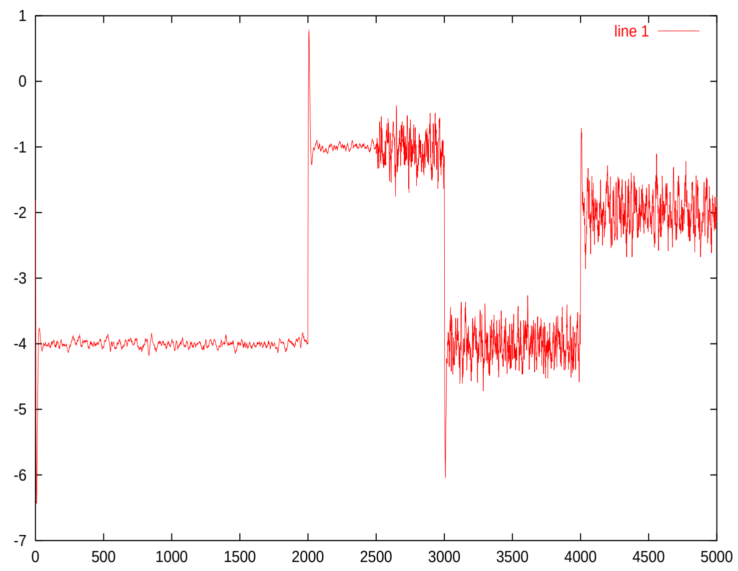
MPC State Estimation Monitoring

- MPC with monitoring Layer



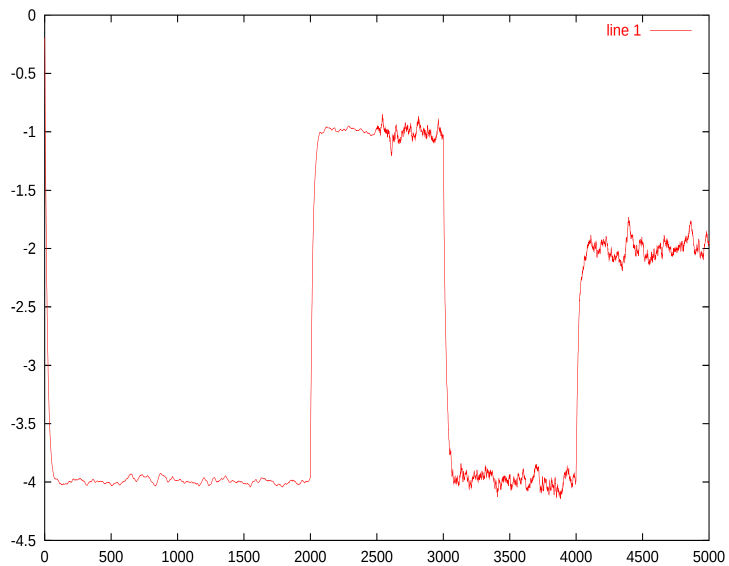
Motivating Example

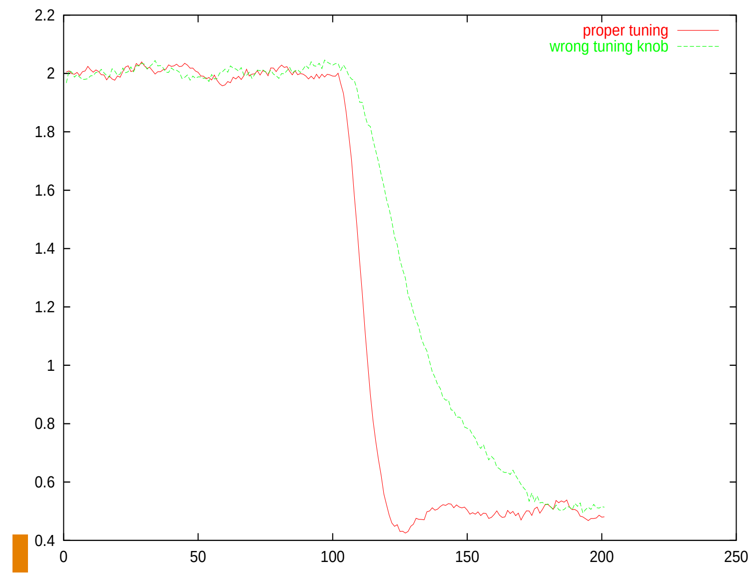
- MPC with sudden increase in sensor noise



Motivating Example

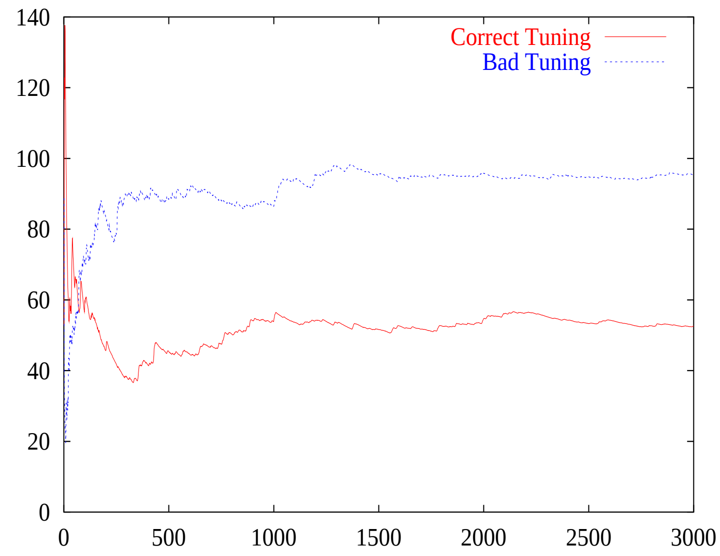
- Operator increases control penalty as a result





Motivating Example

- Objective function may be reduced by correct tuning



Mismatch Effects

Case	Interpretation	Effect	Operator Response	Effect
$R_v \Uparrow$	Sensor quality deteriorates	Excessive control action	Increase control penalty	Slow tracking response
$R_v \Downarrow$	More reliable sensor	Slow tracking response	Probably none	Suboptimal tracking response
$Q_w \Uparrow$	System more variable	Excessive control action	Increase control penalty	Slow tracking response
$Q_w \Downarrow$	System more docile	Slow tracking response	Probably none	Suboptimal tracking response

Stochastic Estimator

$$x_{k+1} = Ax_k + Gw_k \qquad w_k \sim N(0, Q_w)$$
$$y_k = Cx_k + v_k \qquad v_k \sim N(0, R_v)$$

- Perfect model assumption
- Cross Correlations between outputs
- State noise (w_k) propagates through time allowing Q,R separation

- $y_k = Cx_k + v_k$
- $y_{k+1} = CAx_k + Cw_k + v_{k+1}$
- $y_{k+2} = CAAx_k + CAw_k + Cw_{k+1} + v_{k+2}$
- $y_{k+3} = CAAAx_k + CAAw_k + CAw_{k+1} + Cw_{k+2} + v_{k+3}$
- $y_{k+4} = CAAAAx_k + CAAAw_k + CAAw_{k+1} + CAw_{k+2} + Cw_{k+3} + v_{k+3}$

Stochastic Estimator

$$E\begin{bmatrix} y_k y_k^T & & \\ y_{k+1} y_k^T & y_{k+1} y_{k+1}^T & \\ y_{k+2} y_k^T & y_{k+2} y_{k+1}^T & y_{k+2} y_{k+2}^T \end{bmatrix} =$$

$$\begin{bmatrix} CP_k C^T + R & & \\ CAP_k C^T & CAP_k A^T C^T + CQC^T + R & \\ CA^2 P_k C^T & CA^2 P_k A^T C^T + CAQA^T C^T & CA^2 P_k A^{2T} C^T + CAQA^T C^T + R \end{bmatrix}$$

$$E\left[y_j y_k^T\right] = CA^j P_k (A^T)^k C^T + \sum_{h=1}^{k} CA^{j-k+h-1} Q (A^T)^{h-1} C^T + \begin{cases} 0 & j \neq k \\ R & j = k \end{cases}$$

Convergent Solution

- A stable
- k large

$$E\begin{bmatrix} y_k y_k^T & & & \\ y_{k+1} y_k^T & y_{k+1} y_{k+1}^T & & \\ y_{k+2} y_k^T & y_{k+2} y_{k+1}^T & y_{k+2} y_{k+2}^T & \\ y_{k+3} y_k^T & y_{k+3} y_{k+1}^T & y_{k+3} y_{k+2}^T & y_{k+3} y_{k+3}^T \end{bmatrix} =$$

$$= \begin{bmatrix} CSC^T + R & & & \\ CASC^T & CSC^T + R & & \\ CAASC^T & CASC^T & CSC^T + R & \\ CAAASC^T & CAASC^T & CASC^T & CSC^T + R \end{bmatrix}$$

where $S = Q + AQA^T + A^2QA^{2T} + A^3QA^{3T} + \ldots$ (Lyapunov)

Least Squares Problem

- Fit parameters to data

$$E\begin{bmatrix} y_k y_k^T \\ y_{k+1} y_k^T \\ \vdots \\ y_{k+N} y_k^T \end{bmatrix}_{data} = \begin{bmatrix} C & I \\ CA & 0 \\ \vdots & \vdots \\ CA^N & 0 \end{bmatrix} \begin{bmatrix} \widehat{SC}^T \\ R \end{bmatrix}$$

- Care must be taken in setting up LS problem
- LS may try to fit noise
- Use banded window approach to achieve well conditioned problem

Convergent Solution

$$E\begin{bmatrix} y_k y_k^T & & & & & \\ y_{k+1} y_k^T & y_{k+1} y_{k+1}^T & & & & \\ \vdots & y_{k+2} y_{k+1}^T & y_{k+2} y_{k+2}^T & & & \\ y_{k+W} y_k^T & \vdots & y_{k+3} y_{k+2}^T & \ddots & & \\ & y_{k+W+1} y_{k+1}^T & \vdots & \ddots & & \\ & & y_{k+W+2} y_{k+2}^T & \ddots & y_{k+N-2} y_{k+N-1}^T & \\ & & & & y_{k+N-1} y_{k+N-1}^T & y_{k+N} y_{k+N}^T \end{bmatrix} =$$

$$\begin{bmatrix} CSC^T + R & & & & & \\ CASC^T & CSC^T + R & & & & \\ CA^2SC^T & CASC^T & CSC^T + R & & & \\ CA^3SC^T & CA^2SC^T & CASC^T & \ddots & & \\ & CA^3SC^T & CA^2SC^T & \ddots & & \\ & & CA^3SC^T & \ddots & CSC^T + R & \\ & & & & CASC^T & CASC^T + R \end{bmatrix}$$

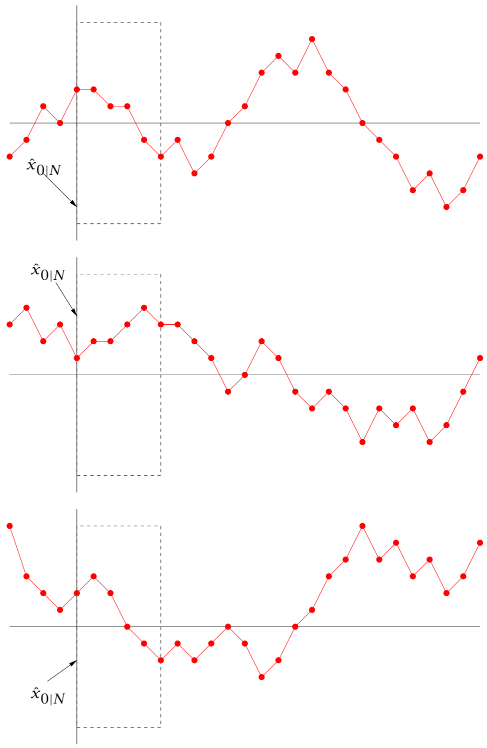
Convergent Solution

$$
E\begin{bmatrix}
y_k y_k^T \\
y_{k+1} y_k^T \\
y_{k+2} y_k^T \\ \hline
y_{k+1} y_{k+1}^T \\
y_{k+2} y_{k+1}^T \\
y_{k+3} y_{k+1}^T \\ \hline
\vdots \\ \hline
y_{k+N-2} y_{k+N-1}^T \\
y_{k+N-1} y_{k+N-1}^T \\
y_{k+N} y_{k+N-1}^T \\ \hline
y_{k+N} y_{k+N}^T
\end{bmatrix}
=
\begin{bmatrix}
C & I \\
CA & 0 \\
CAA & 0 \\ \hline
C & I \\
CA & 0 \\
CAA & 0 \\ \hline
\vdots & \vdots \\ \hline
C & I \\
CA & 0 \\
CAA & 0 \\ \hline
C & I
\end{bmatrix}
\begin{bmatrix}
\widehat{SC}^T \\
R
\end{bmatrix}
$$

1. Build least squares problem
2. Build windows to achieve observability (if possible)
3. Compute expectations from data
4. Compute R and SC^T
5. Extract Q from SC^T

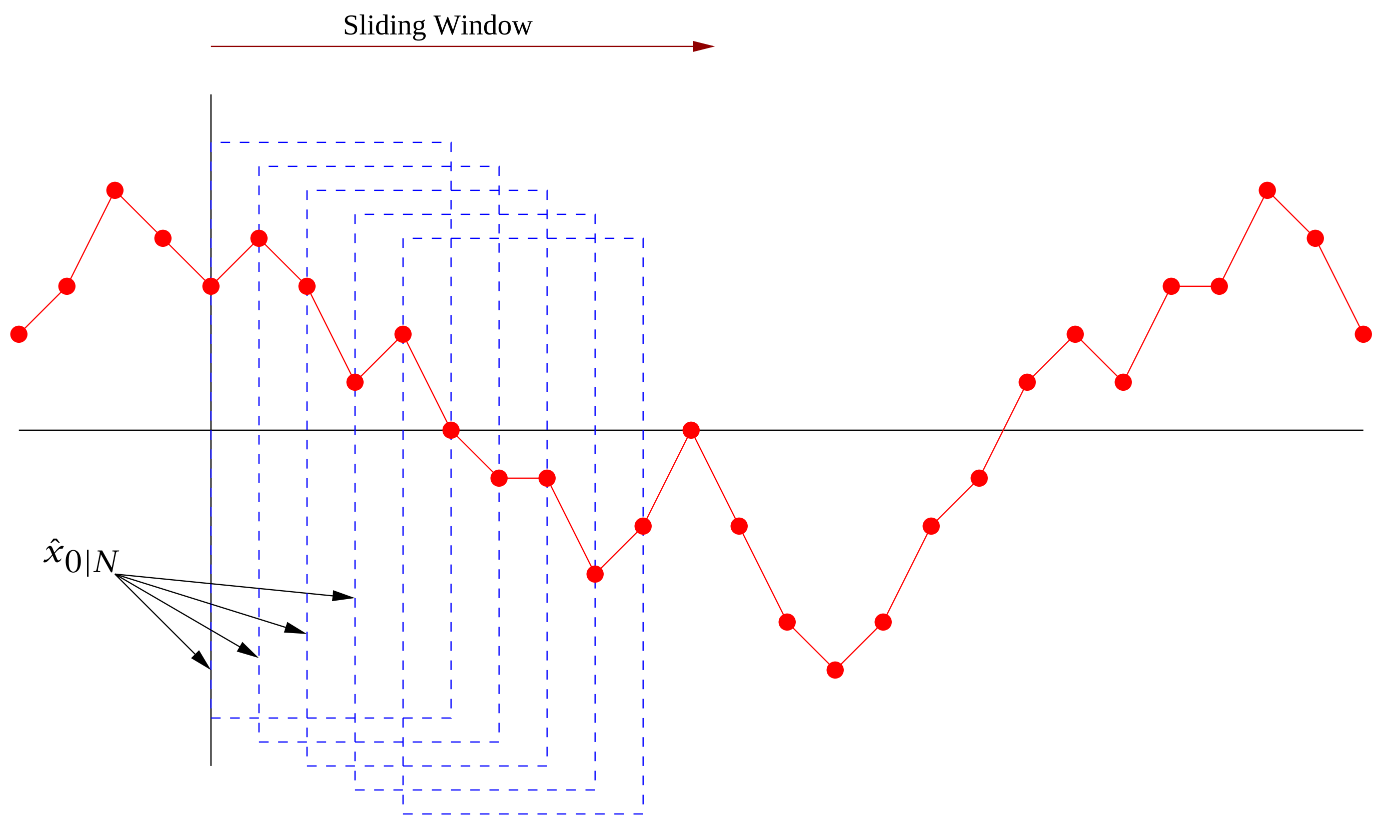
Computing Expectations

- Replicate experiments provide good data
- Not practical on a real process



Computing Expectations

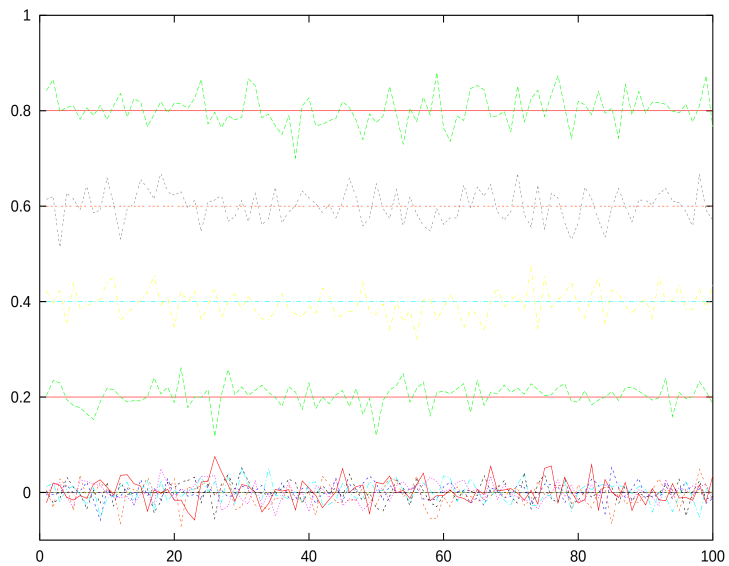
- Better to use sliding window to provide “replicates”



Open Loop Examples

$$R = \begin{bmatrix} 0.2 & & & \\ 0 & 0.4 & & \\ 0 & 0 & 0.6 & \\ 0 & 0 & 00.8 & \end{bmatrix}$$

- Element by element estimation of R



Closed loop modifications

1. Augment System for stability
2. Use augmented system for MPC model to apply constraints
3. Subtract constraining control actions to obtain unconstrained outputs
4. Apply "open loop" methodology

$$u_k = \underbrace{\begin{bmatrix} K_x & K_u \end{bmatrix} \begin{bmatrix} \hat{x}_k - x_k^s \\ u_{k-1} - u_k^s \end{bmatrix} + u_k^s}_{\text{stabilizing/tracking part}} + \underbrace{p_k}_{\text{constraining part}}$$

Closed loop modifications

$$z_{k+1} = \tilde{A} z_k + \tilde{B} p_k + \tilde{G} \tilde{w}_k$$
$$y_k = \tilde{C} z_k + v_k$$

$$\tilde{w} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \right)$$

$$v \sim N(0, R) \tag{1}$$

- Closed loop methods build in cross-correlation terms

$$E \begin{bmatrix} \tilde{w}_k \\ v_k \end{bmatrix} \begin{bmatrix} \tilde{w}_k^T & v_k^T \end{bmatrix} = \left[\begin{array}{cc|c} Q & 0 & 0 \\ 0 & R & R \\ \hline 0 & R & R \end{array} \right] \tag{2}$$

Closed loop Example

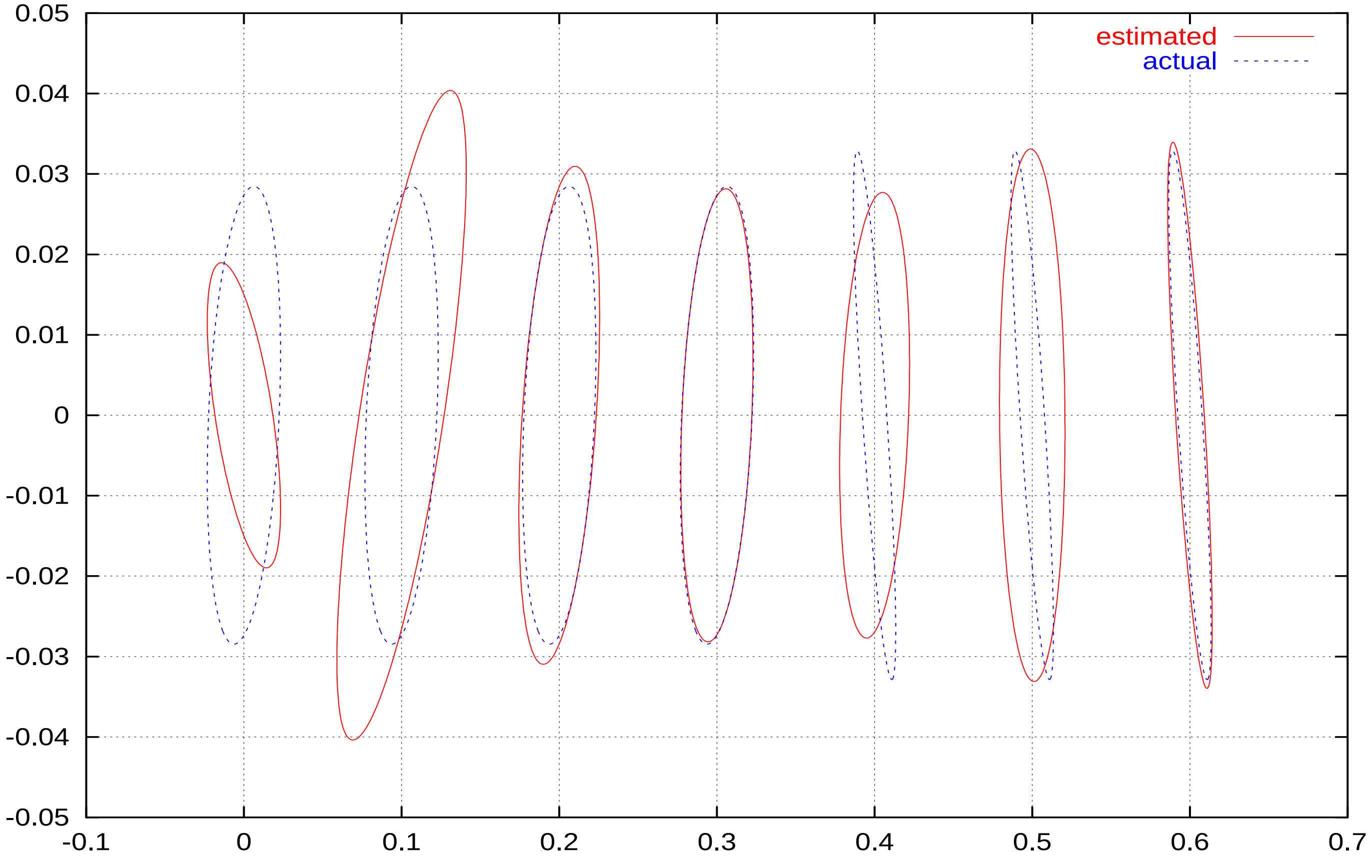
$$G(z) = \begin{bmatrix} \frac{z}{2z-1} & \frac{z}{2.5z-1.5} \\ \frac{0.5z}{2z-1} & \frac{1.5z}{2.5z-1.5} \end{bmatrix}$$

$$A = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.6 \end{bmatrix} \quad B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \\ 0.25 & 0 \\ 0 & 0.6 \end{bmatrix}$$

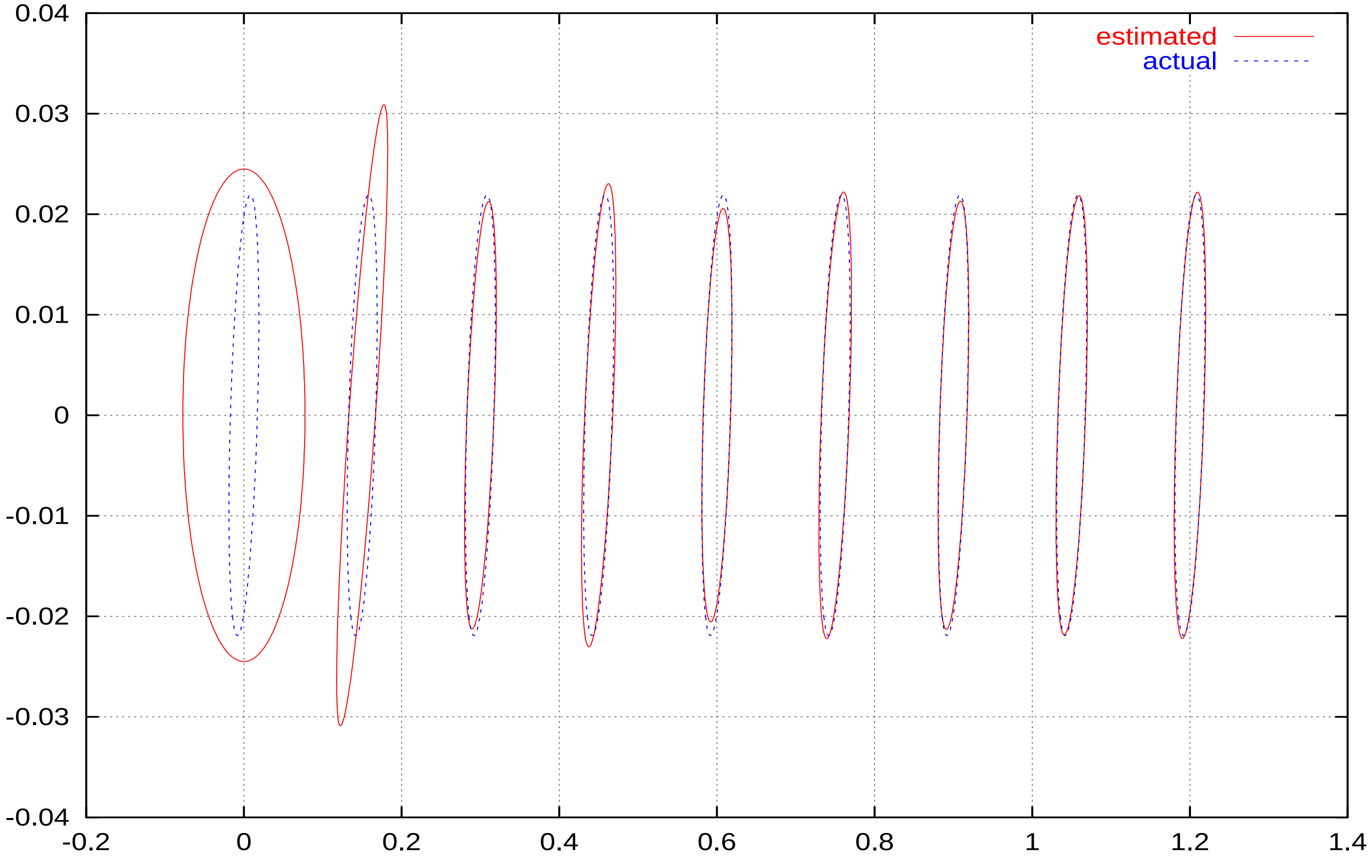
$$C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.006 & 0.002 \\ 0.002 & 0.009 \end{bmatrix} \quad \widehat{R} = \begin{bmatrix} 0.006 & -0.003 \\ -0.003 & 0.004 \end{bmatrix}$$

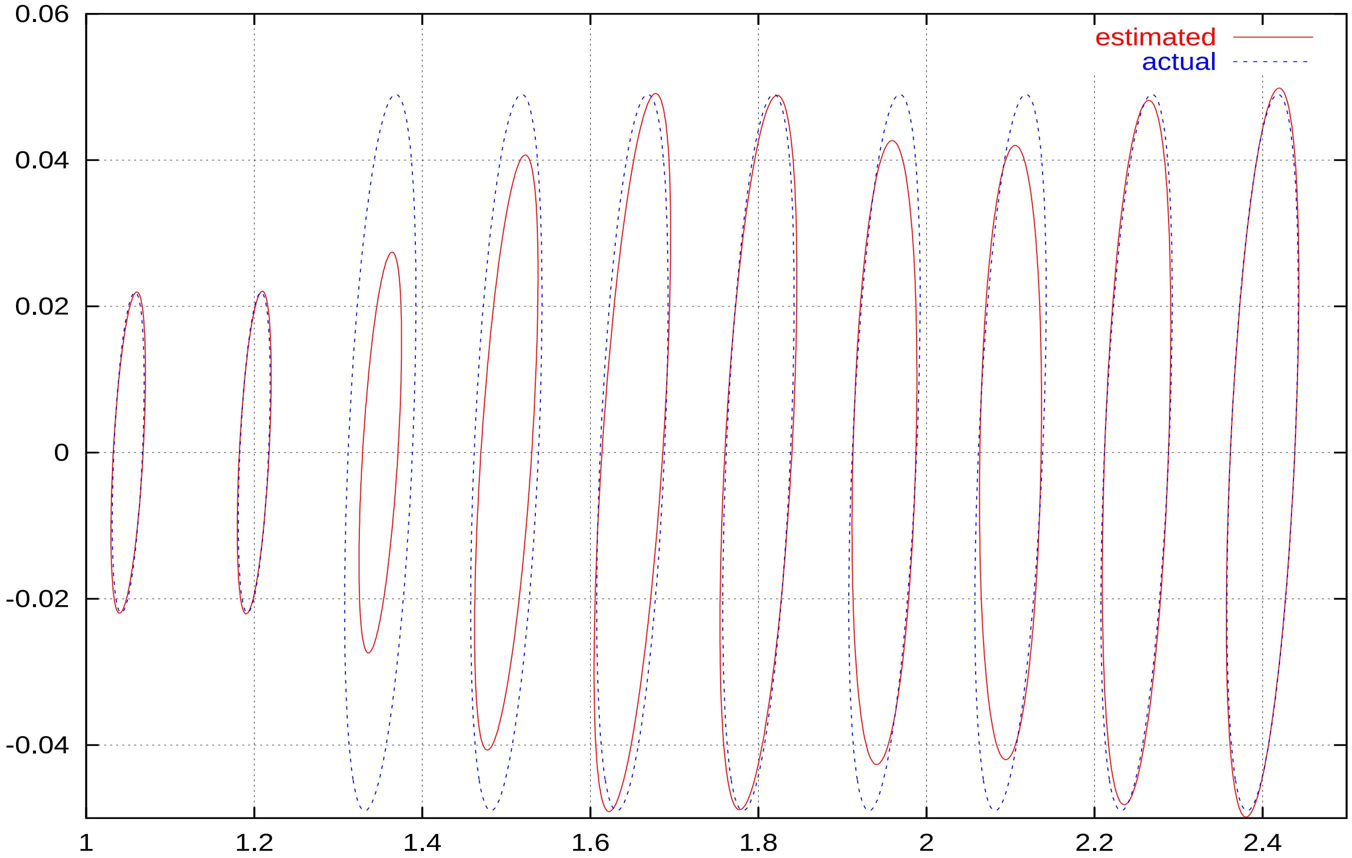
Closed loop Example



Initial Tuning



Dynamics Shifts



Shell Control Example

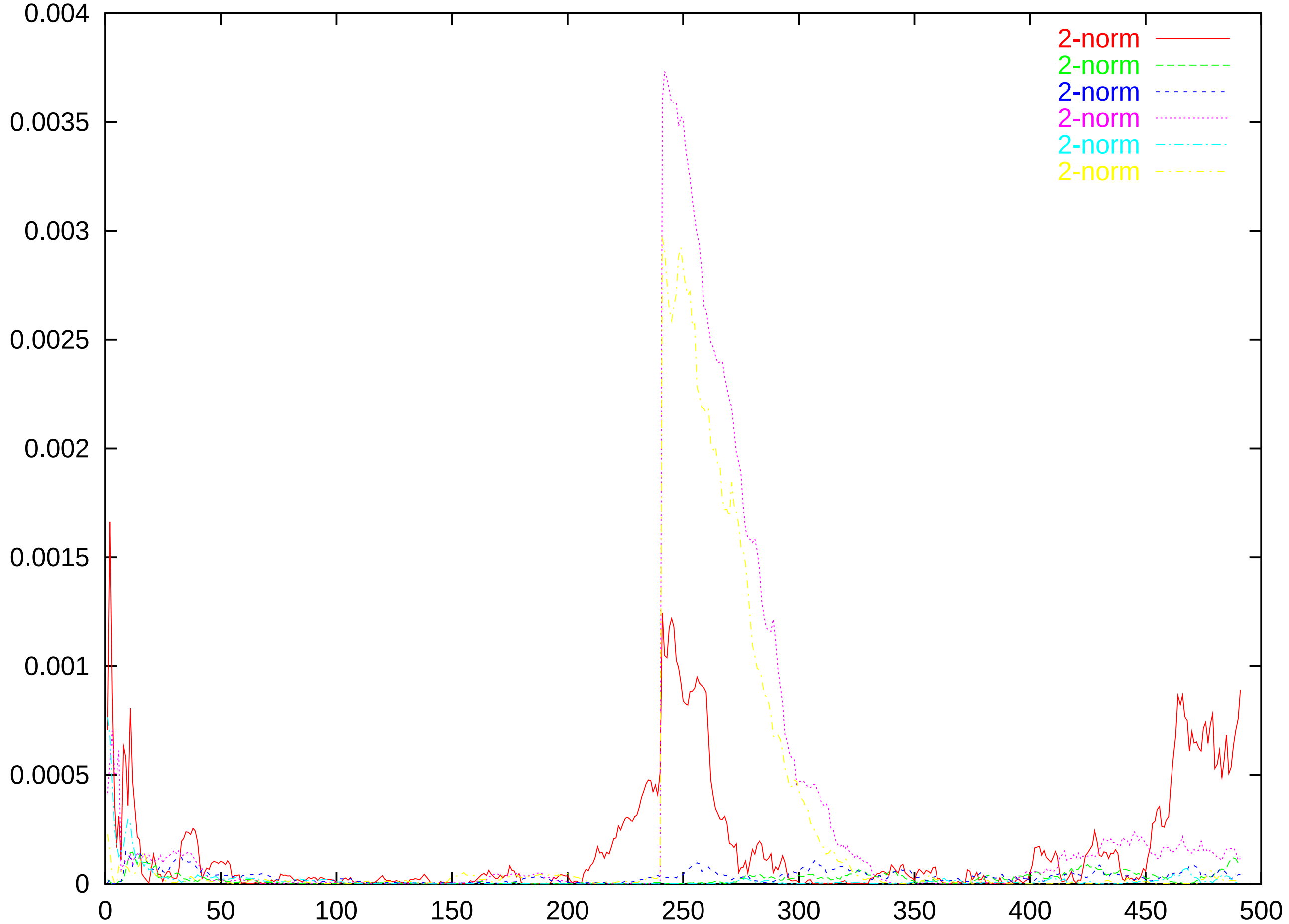
$$G(s) = \begin{bmatrix} \frac{4.05e^{-27s}}{50s+1} & \frac{1.77e^{-28s}}{60s+1} & \frac{5.88e^{-27s}}{50s+1} \\ \frac{5.39e^{-18s}}{50s+1} & \frac{5.72e^{-14s}}{60s+1} & \frac{6.90e^{-15s}}{40s+1} \\ \frac{4.38e^{-20s}}{33s+1} & \frac{4.42e^{-22s}}{44s+1} & \frac{7.20}{19s+1} \end{bmatrix}$$

$$-0.5 \leq u \leq 0.5$$

$$y_t = \begin{bmatrix} 0.3 \\ 0.3 \\ -0.3 \end{bmatrix} \text{(unreachable)}$$

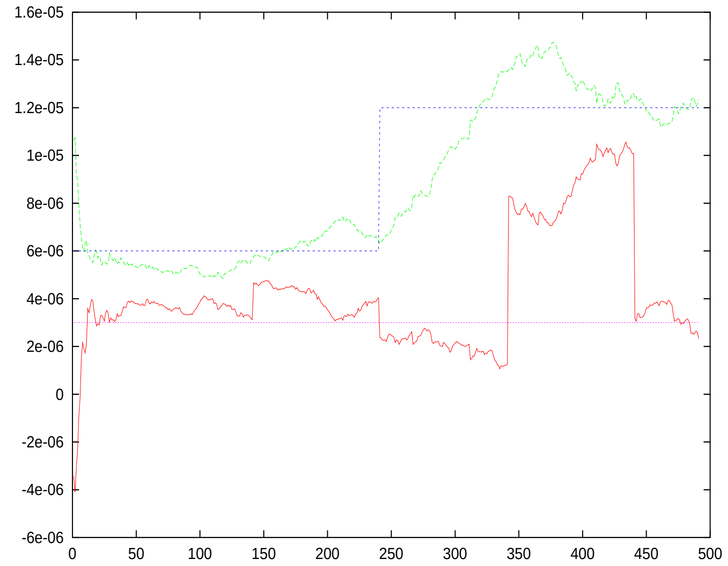
- 30 states
- 12 Covariance elements to estimate

Shell Control Example



Output disturbances

$$x_{k+1} = Ax_k + Bu_k + Gw_k$$
$$y_k = Cx_k + d_k + v_k \quad (3)$$

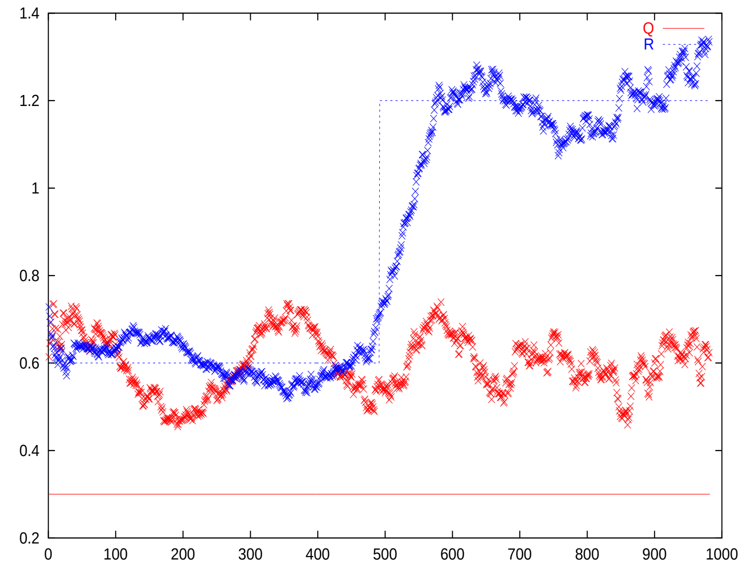


- Disturbances affect estimation monitoring intially

Robustness

$$g(s) = \frac{1}{(s+1)(s+2)} \tag{4}$$

- 20% mismatch in time constant

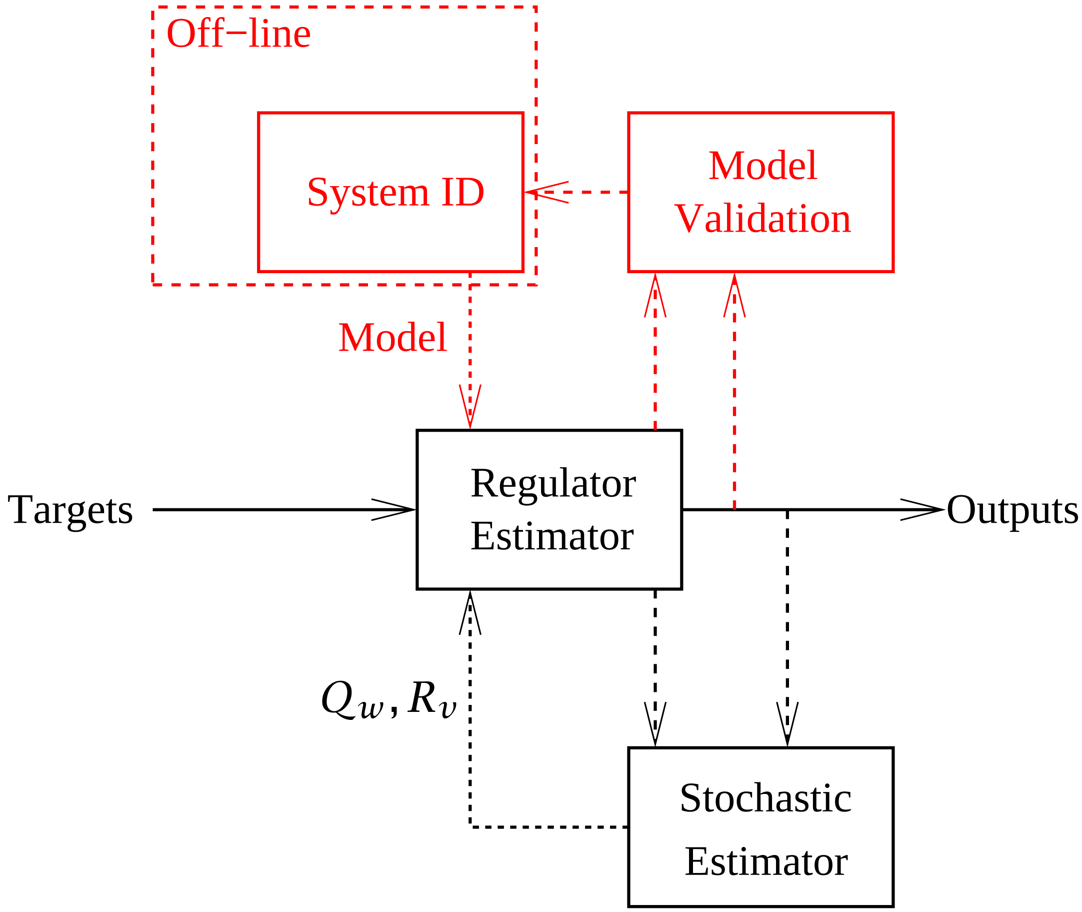


$$Q = \begin{bmatrix} 0.3 \end{bmatrix}$$
$$R = \begin{cases} 0.6 & t \leq 500 \\ 1.2 & t > 500 \end{cases}$$

- Model mismatch leads to incorrect estimates

- Yields optimal tuning for incorrect model, however

Model Identification



Overall Strategy

1. Initial on-line validation of model

2. Re-identify model until confidence reached

3. Begin online estimation of tuning parameters

4. Correct tuning if constraints violated

5. Periodic testing for validity of model

Conclusions:

1. Q_w and R_v can be identified from data in many cases

2. Estimation is subject to model mismatch, but not in a catostrophic way

Future Work

Short Term

- Test methodology with data
- Downs problem (startup monitoring)
- Configure module with different MPC strategies

Long Term

- Implement model mismatch detection
- Combine package into robust industrial software

Acknowledgements

- Jim Rawlings
- John Eaton
- Daniel Patience