Feasible Real-time Moving Horizon Estimation

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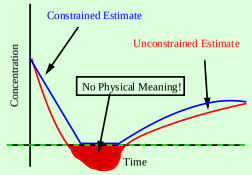
1 Motivation

In state-space model predictive control (MPC), it is necessary to recreate estimates of the states from measured outputs. We present moving horizon estimation (MHE) as an alternative to the popular extended Kalman filter (EKF) for the following reasons:

- Solves minimization problem to fit data.
- Incorporates constraint handling.
- Possesses stability properties.

2 Estimator Constraints

Constraints are used to prevent the estimator from returning results with no physical meaning, such as negative temperatures or concentrations, or mass and energy balance violations such as negative material loss.



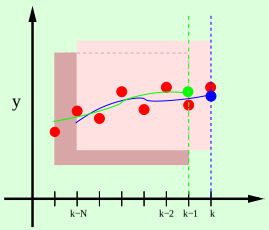
3 MHE Formulation

$$\min_{\{w_k\},\{\hat{x}_k\},\{v_k\}} \rho^T P \rho + \sum_{j=0}^{N-1} \left[w_j^T Q w_j + v_j^T R v_j \right] + v_N^T R v_N$$

subject to:

$$\hat{x}_0 - \bar{x}_0 + \rho$$
$$\hat{x}_{k+1} - F(\hat{x}_k, u_k, w_k, t)$$
$$y_k - g(\hat{x}_k) + v_k$$
$$Hx_k \le h, \qquad Sw_k \le s, \qquad \Gamma v_k \le \gamma$$

Given a set of data, the estimate at time k-1 (green) is updated to the estimate at time k (blue).



4 Structured SQP

- We optimize using a sequential quadratic programming (SQP) technique, which has the following properties:
  - Solves a series of quadratic programs (QPs) which approximate nonlinear problem at the current iterate.
  - Each QP has a quadratic objective function and linear constraints.
- We focus on feasibility of each quadratic program. This requirement will allow us to run the estimator suboptimally, if desired.
- We solve each QP with a specially structured solver, which exhibits linear growth with horizon length. Normal solvers grow cubically.

The structure of the KKT conditions to be solved is:

$$\begin{bmatrix} -I & M_k & & C_k^T & & A_k^T & \\ & R_k & I & & & & \\ C_k & I & & & & & \\ & & & Q_k & G_k^T & & \\ A_k & & & G_k & & -I & \\ & & & & -I & \Pi_k & \end{bmatrix} \begin{bmatrix} \Delta p_{k-1} \\ \Delta x_k \\ \Delta v_k \\ \Delta q_k \\ \Delta w_k \\ \Delta p_k \\ \Delta x_{k+1} \end{bmatrix} = - \begin{bmatrix} \tilde{r}_k^x \\ \tilde{r}_k^v \\ \tilde{r}_k^q \\ \tilde{r}_k^w \\ \tilde{r}_k^p \\ \pi_k \end{bmatrix}$$

This system of equations is solved by an iterative procedure:

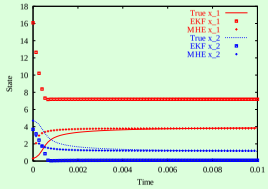
$$\Pi_{k-1} \leftarrow -M_k + C_k^T R_k L_k^q + A_k^T \Pi_k A_k - A_k^T \Pi_k G_k \left(Q_k + G_k^T \Pi_k G_k \right)^{-1} G_k^T \Pi_k A_k$$
$$\pi_{k-1} \leftarrow -\tilde{r}_k^x + C_k^T R_k \tilde{r}_k^q - C_k^T \tilde{r}_k^v + A_k^T \Pi_k \tilde{r}_k^p + A_k^T \pi_k$$
$$- A_k^T \Pi_k G_k \left(Q_k + G_k^T \Pi_k G \right)^{-1} \left[G_k^T \Pi_k \tilde{r}_k^p + G_k^T \pi_k + \tilde{r}_k^w \right]$$

5 MHE vs. EKF

Consider the following simple mathematical example with initial estimate error:

$$\dot{x}_1 = k_1 x_1^2 x_2^5 - k_2 x_1^3 x_2^4$$
$$\dot{x}_2 = -k_1 x_1^2 x_2^5 + k_2 x_1^3 x_2^4$$

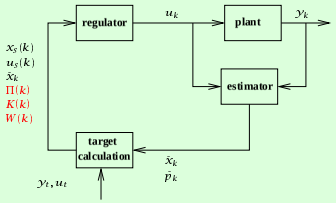
and output $y = x_1^4 x_2$.



⇒ Even with a correct model, the EKF converges to the wrong state estimate!

6 MHE in a Closed Loop

We can incorporate MHE into a complete model predictive controller by applying it as the estimator in the following system:



The regulator has the following properties:

- Can function as a feasible suboptimal controller.
- Uses efficient structured QP solver.

7 Copolymerization Process

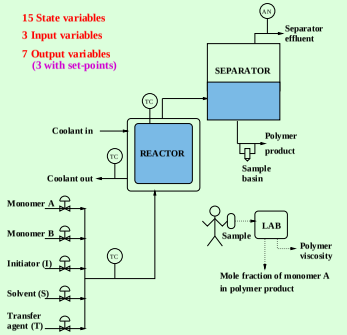


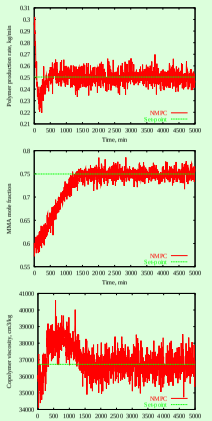
Figure 1: MMA-VA Copolymerization System

Grade	Polymer production rate (kg/min)	MMA Mole fraction in copolymer	Copolymer viscosity ($10^{-6} m^3/kg$)
A	0.3	0.60	35000
B	0.25	0.75	36725

Table 1: Product grades for copolymerization

8 Grade Transition

We investigate a change from Grade A to Grade B in which the estimated initial state is incorrect in all variables by 5% and normally distributed measurement noise with a standard deviation of 2% is present.



9 Conclusions

- MHE can converge to the true states when EKF cannot.
- Our method incorporates:
  - Efficient structured solution method for regulator and moving horizon estimator
  - Nonlinear moving horizon estimator with integrating disturbance model
  - Nonlinear steady-state target calculation
- The combined controller and estimator run in real-time for industrially important processes.

10 Future Work

- Smoothing covariance update for MHE.
- Structured Hessian updates.
- Make NMPC tool available on internet.