Robust Model Predictive Control

Yiyang (Jenny) Wang

University of Wisconsin - Madison, Department of Chemical Engineering

Research Objective

- Develop a new robust MPC theory that stabilizes a system with a polytopic model uncertainty description Ω. $(A, B) \in \Omega$ if

$$A = \sum_{i=1}^{I} \mu_i A_i, \quad B = \sum_{i=1}^{I} \mu_i B_i, \quad \sum_{i=1}^{I} \mu_i = 1, \quad \mu_i \geq 0$$

- Add a tree trajectory to the regulator to simulate time-varying uncertainties.
- Modify the controller formulation to include offset free non-zero set point tracking.
- Develop and evaluate the closed-loop stability conditions when model uncertainty is present.

Integral Control

Without Target Calculation

- The process models are Auto Regressive Moving Average Exogenous Inputs (ARMAX) polytopic models.
- The state is defined as

$$x_k = [y_k^T, \ldots, y_{k-n-1}^T, u_{k-n-1}^T, \ldots, u_{k-1}^T \epsilon_k^T]^T$$

- Integral control is achieved by taking control action on ϵ_k.

$$\epsilon_k = \sum_{j=0}^{k} (y_t - y_j)$$

- K_1 and K_2 are the feedback gains on the extended state and ϵ_k, respectively.
- Offset free integral control is achieved when $\epsilon_k \longrightarrow 0 \implies y_k \longrightarrow y_t$.

With Target Calculation

- Add disturbance models to achieve integral control.

$$x^{\{\tau_j, i\}}(k) = A_i x^{\{\tau_j\}}(k) + B_i u^{\{\tau_j\}}(k) + p_i(k)$$

- Determine a single (x_s, u_s) pair for all models such that the deviation between Cx_s and y_t is minimized.

$$x_s = A_i x_s + B_i u_s + p_i \qquad \forall i = 1, \ldots, I$$

Suspension System

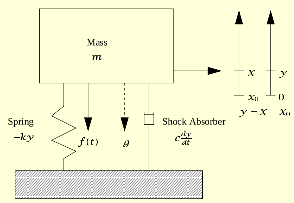


Figure 1: The spring-shock absorber system.

The parameters of the suspension system are:

spring constant, k = 2000 newtons/m
mass of load, m = 500 kg
amplitude of test force, A = 50 newtons
shock absorber constant, c = 400 newtons/(m/s)

The suspension system model for deviation from the desired position x_0, y, is given by:

$$m\frac{d^2y}{dt^2} = -ky - c\frac{dy}{dt} + f(t)$$

in which

$$\tau = \sqrt{\frac{m}{k}} \qquad K = \frac{1}{k} \qquad \zeta = \sqrt{\frac{c^2}{4mk}}$$

Three different process models are developed by sampling the system's open-loop data at three different sampling rates.

1. $\Delta t = \tau/10$ 2. $\Delta t = \tau/5$ 3. $\Delta t = \tau/2$

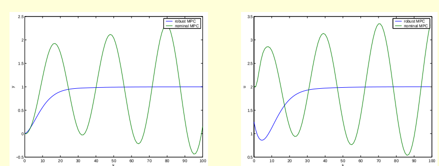


Figure 2: Plant is model 2 but nominal MPC is using model 3

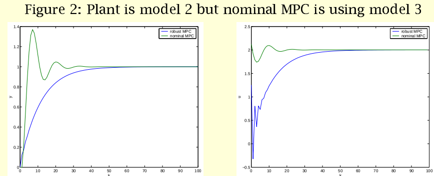


Figure 3: Plant is model 3 and nominal MPC is using model 3

Suspension System

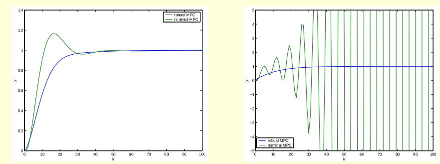
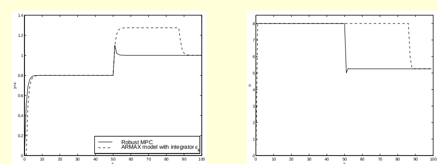


Figure 4: Plant is model 2 for the left figure and model 3 for the right figure. Nominal MPC is using model 1

Constraint Saturation Example

$$G_1 = \frac{0.1}{s+10} \qquad G_2 = \frac{1}{s+10}$$

$$y(s) = G_i u(s)$$
$$-\infty \leq u_k \leq 8$$
$$-\infty \leq y_k \leq \infty$$



Remarks:

- Model 2 is the plant and $y_t = 1$.
- The input constraint causes the set point to be unreachable.
- A disturbance at time $k = 50$ enters the system. The set point is reachable.
- Both robust MPC with target calculation and integral control with ARMAX models successfully reject the disturbance and reach the set point.
- But integral control with the ARMAX model exhibits windup behavior by delaying the decrease of u_k from its constraint even though the set point is reachable as soon as the disturbance enters the system.

Infinite Horizon Regulator

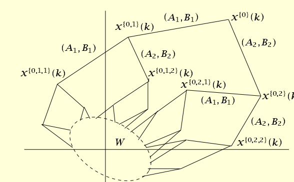


Figure 5: Regulator tree trajectory for $I = 2$ and $N = 4$.

$$\min_{\pi^l(k)} \max_{l=1,\ldots,\mathcal{L}} \Phi^l(k)$$

subject to

$$x^{\{\tau_j, i\}}(k) = A_i x^{\{\tau_j\}}(k) + B_i u^{\{\tau_j\}}(k)$$
$$x^{\{\tau_j\}}(k) \in W \ \forall j \geq N, \quad x^{\{\tau_j\}}(k) \in X, \quad u^{\{\tau_j\}}(k) \in U$$

Remarks:

- $\Phi^l(k)$ is the performance objective for trajectory l at time k.

$$\Phi^l(k) = x^{\{\tau_N\}^T}(k) F x^{\{\tau_N\}}(k) + \sum_{j=0}^{N-1} x^{\{\tau_j\}^T}(k) Q x^{\{\tau_j\}}(k) + u^{\{\tau_j\}^T}(k) R u^{\{\tau_j\}}(k)$$

- W is the invariant terminal region in which the control policy $u^{\{\tau_j\}}(k) = Kx^{\{\tau_j\}}(k)$ is robustly stabilizing.
- $F = F^T > 0$ is the final state penalty matrix.

$$F = \sum_{j=N}^{\infty} x^{\{\tau_j\}^T}(k) Q x^{\{\tau_j\}}(k) + u^{\{\tau_j\}^T}(k) R u^{\{\tau_j\}}(k)$$

in which $Q \geq 0$ and $R > 0$.

- $\mathcal{L}$ is the maximum number of possible trajectories, $\mathcal{L} = I^N$.
- $(A, B) \in \Omega$ is robustly stabilizable if there exists K and F such that

$$F - Q - K^T RK - (A_i + B_i K)^T F (A_i + B_i K) \geq 0 \quad \forall i = 1, \ldots, I$$

Conclusion

- Developed the robust MPC theory that controls a time-varying uncertain system described by $(A, B) \in \Omega$.
- Modified the theory to include integral control that achieves offset free non-zero set point tracking without exhibiting windup in the presence of constraint saturation.
- Developed stability conditions that guaranteed offset free performance.